

Zipped Assembly

by

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B.Sc., Massachusetts Institute of Technology (2015)

Submitted to the Program in Media Arts and Sciences, School of Architecture and Planning, in  
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Master of Science

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## Abstract:

Biology creates assemblies with orders of magnitude more parts than any known human designed process. Molecular biology functions on the premise that fundamental building blocks assemble into chains, are zipped into strands and folded into structures. This thesis is a macroscale implementation that aims to do the same, assemble, zip and fold, in an inorganic system. This system, Zipped, utilizes distributed coalescence of parts, aiming for faster assembly while incorporating error correction into the fabrication process.

This thesis presents a design for 0-dimensional building blocks that snap together to form 1-dimensional strands. Strands zip together, interlocking to form 2-dimensional beams that can branch and merge to create patterns or flat sheets. Strands can zip to each other out of plane as well, allowing 3-dimensional construction. All steps of the construction process are reversible; parts can be assembled, dis-assembled and re-assembled without damage to the part or altering structural performance. No energy, formwork or pre-load is required to maintain the parts position once it is assembled.

The system can assemble rigid as well as flexural elements, including chains and revolute joints. Increased stiffness or flexibility can be designed into structures by changing strand geometry and zipping. This ability to tune local structural properties allows actuators to be added to the construction system and form mechanisms. Zipped pieces are demonstrated as the structural element for a robot's body, which can locomote on itself or foreign terrain. Initial studies also demonstrate automated construction with this system.

The fundamental principles of this system are demonstrated in many materials, via different manufacturing processes and across several scales, showing applicability to a diverse scenario space. For ease of fabrication and lab use a centimeter scale part was selected and several thousand parts were manufactured. This 0-dimensional part is presented and used to form larger scale assemblies which are mechanically characterized. From here, mission architectures and real-world applications are described. The Zipped system enables human-scale, controlled and reversible assembly, zipping and folding. This allows reusability, reconfigurability and universality - attributes we often credit to nature.

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# 1 INTRODUCTION

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Development of a modular, autonomously reconfigurable robotic system has long been roboticists' dream. There are many examples of systems that morph from one structure into another depicted in science fiction, but nothing in the real world has yet matched these visions. Modular, autonomously reconfigurable structures and systems would have many applications in infrastructure and aerospace, but research has yet to merge the notions of programmable material and robotic assembly at scales larger than 1m. Modular robotic systems, where the robot itself transforms, tend to have low stiffness to weight ratios and thus struggle to scale the size of construction. Robotic assembly systems, where the robot builds using a different material, tend to tradeoff structural performance for constructability.

On a much smaller unit scale than mechanical parts, biology does mirror these science fiction systems. Pictured below is a diagram showing protein synthesis as amino acids form primary, secondary, tertiary then quaternary structures [1]. Below it Microbots from Big Hero 6 are shown [2]. On the left is a single unit which, as is mentioned in the film, can't do much. In the middle several units are joined to make a bi-pedal robot that can move and fight, and on the right several thousand bots are depicted creating massive, moving structures that carry people as fast as cars. Though there are many limiting factors between the animated representations, biological replication and state of the art robots, this thesis presents an approach to push the boundary toward science fiction. Using a chain-based joining mechanism between parts a load-bearing, structural system was designed which assembles via a process mirroring biological fabrication. This system transforms into different architectures and can hold shape without consuming energy, allowing actuators to be mobile, traveling along the structure building and repairing it.

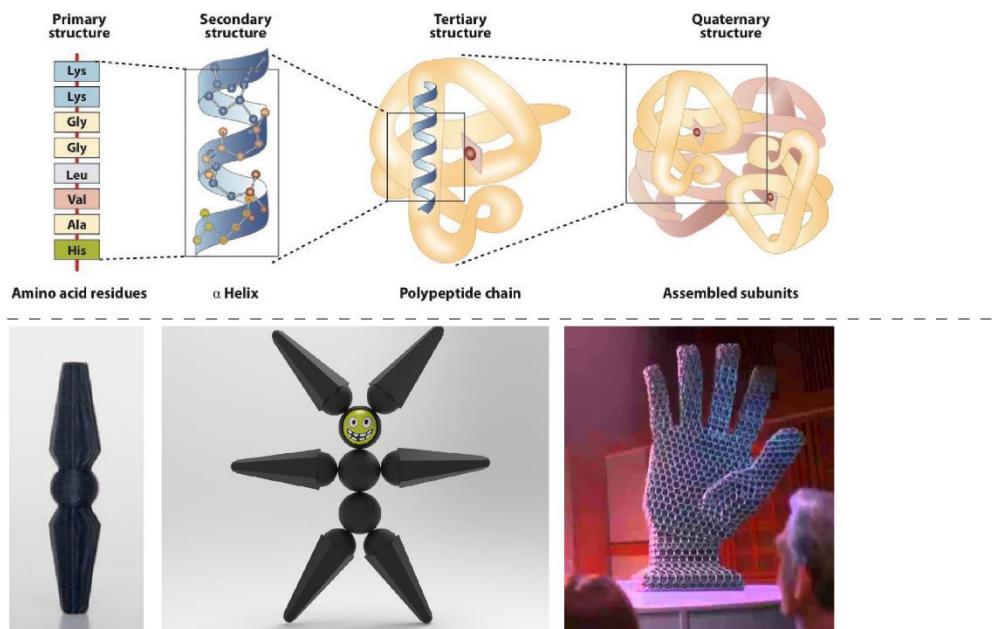


Figure 1 Comparison of the Microbots in Big Hero 6 to protein synthesis. Image credit: [1], [2]

## 1.1 ASSEMBLY

Though assembly has been readily adopted in industry from the introduction of interchangeable parts in the 19<sup>th</sup> century, which allowed the industrial revolution [3], very few products are designed for disassembly. In 2019 consumer electronics was a 300 billion dollar industry [4] which is built upon a handful of components that tile into assemblies that are soldered together. Permanently binding these assemblies causes growing environmental problems [5]. As human beings in a world of limited resources, it is imperative we start designing more sustainable systems and increase material recycling and reuse.

In addition to the environmental impact, assembly can allow personalized systems, such as presented by Google's Project Ara [6], which allowed users to upgrade smart phone modules without purchase of an entirely new device. Previous work has shown that mechanical assembly processes can create objects more precise than the machine assembling them and projections indicate assembly, as seen in biology, will exponentially increase speed of fabrication, made possible by recursive assembly, where parts are able to assemble the machine that assembled themselves [7].

Current modules of assembled systems however, be it machine parts, smart phones, or packages housing semi-conductors, are all still highly complex subsystems, often consisting of multiple materials and fabricated through several different manufacturing processes. In contrast, all of life is assembled from 21 amino acids at speeds and resolutions orders of magnitude higher than any human derived manufacturing technique [8]. Key to mimicking nature is rapid mass manufacture of an affordable building block that can assemble a wide variety of things.

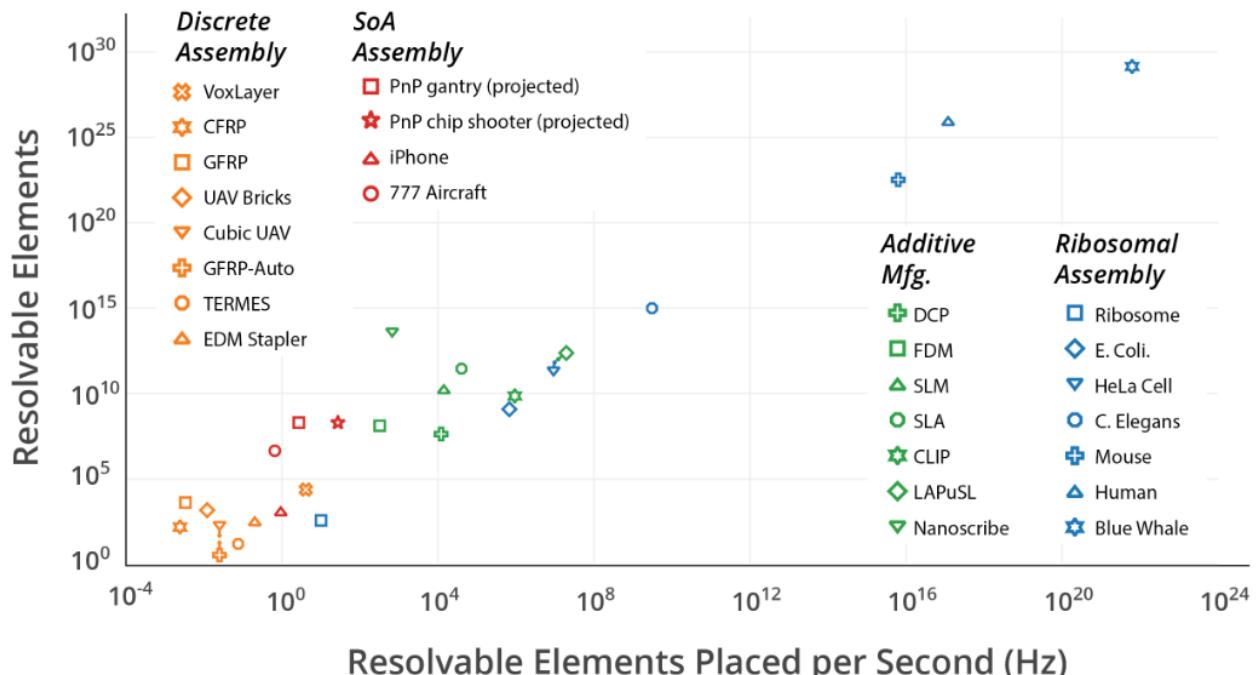


Figure 2 Manufacturing throughput vs dynamic range across manufacturing processes. Biology's throughput scales with size enabling the fabrication of organisms with far more fabrication complexity than any state-of-the-art technological. Image credit: Will Langford [7]

In manufacturing, tolerances allow parts to be produced in batches, increasing output and decreasing price, while still maintaining their functionality and fit within the assembly. A classic example of impressive manufacturing tolerances is provided by Lego, where parts fit together across decades and different manufacturing facilities [9]. The system produced in this thesis is similar in scale and can fulfil similar functions as Lego, both being assembly construction sets. Zipped differs in its ability to form flexible walls and chains, which Lego blocks can't inherently do. Lego is predominantly indented for bulk, rectilinear construction, whereas other systems, including PinBlock [10], Better Blocks [11] and Nanoblock [12], are designed to assemble flexible surfaces. Each block for these toys has two pins connecting it to its neighbors acting as revolute joints, allowing a wall of blocks to be stacked, yet still flex and bend. However, these systems are not capable of rigid construction. Zipped fills this gap by offering an assembled construction system that can create both flexible and rigid structures.



Figure 3 Better Blocks, Nanoblocks and PinBlocks. Three examples of toy construction systems where the connection between blocks is a revolute joint. Image credit: [10], [11], [12]

At the architectural scale, assembly is perhaps the most common form of fabrication, including timber construction, masonry and pre-fabricated housing. Topologically interlocked materials (TIMs) also fall into this category. TIMs are designed building blocks that fit together, tessellating in 3-dimensions to create load-carrying assemblies. These blocks are constrained in space such that none can be removed from the structure, excluding those on the periphery. Typically, TIMs assemblies require formwork to hold position. They can be bound by a frame or pre-loaded with tension cables to constrain the assembly [13]. TIM geometries include platonic solids, osteomorphic and mating concave-convex geometries, which can be created by lofting tessellations. Examples are shown in the figure below. TIMs have been shown to allow tunable mechanical properties to be assembled within a structure, such as bending stiffness and load-bearing capacity, to have high sound and impact energy absorption, and to be robust to local failures, in addition to ease of assembly and dis-assembly [14],[15].

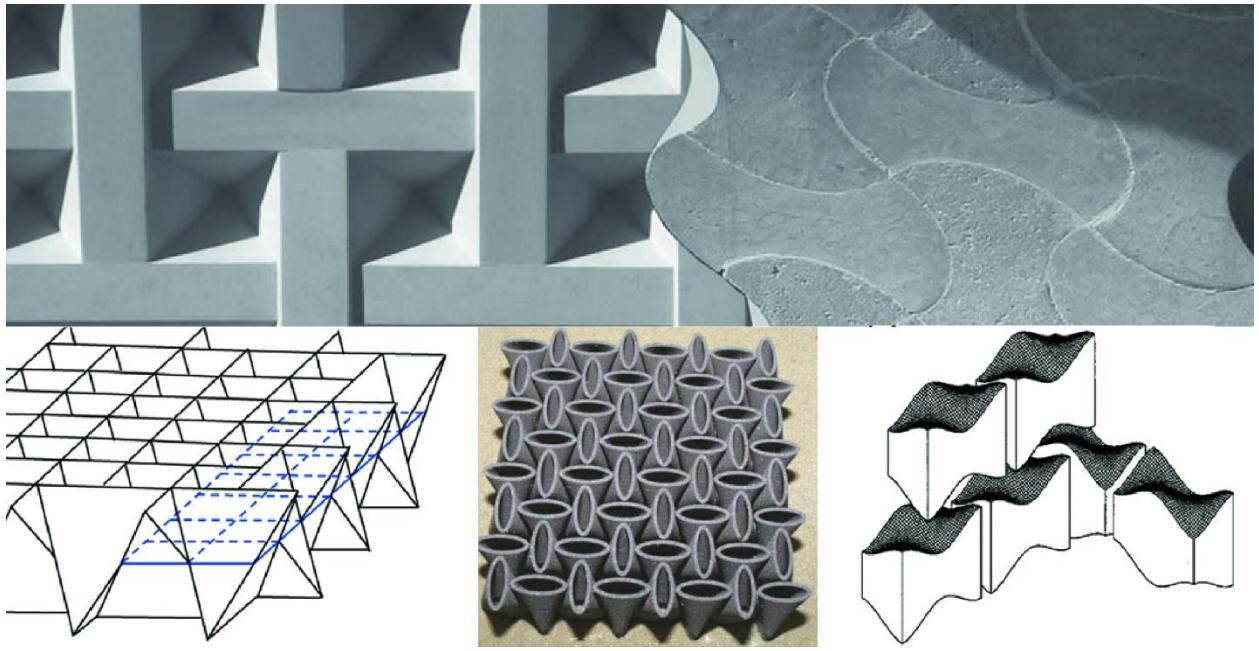


Figure 4 Various topologically interlocked materials. Image credit: [15]

## 1.2 ZIPPER BACKGROUND

A zipper is a mechanism used to connect the edge of two surfaces. Elias Howe was the first to patent a zipper-like fastener in 1851. In 1893, Whitcomb Judson designed a similar fastener he called a “clasp locker” [16]. Early zipper prototypes, pictured below, had claw-like teeth that locked into a chain link. In 1917 Judson and Sunback patented the modern zipper, but it wasn’t commonly seen in clothing until the 1930’s [17], [18]. For reference, DNA was discovered in 1869 and in 1953 Watson and Crick concluded the DNA molecule forms a double helix [19].



Figure 5 Zipper circa 1850, modern zipper, DNA replication graphic. Image credit: [20], [21], [22]

The geometry from the first zippers can also be seen in modern mechanisms as it provides a stable, mechanical connection. These implementations are intended for applications which require flexible to rigid mechanical properties, as opposed to the modern zipper which is intended as a closure mechanism and would ideally remain flexible while zipped or un-zipped. Pictured below are figures of a long throw linear actuator, a rigid beam that can be erected and deconstructed. These diagrams were taken from TSUBAKIMOTO CHAIN CO.'s patent [23] on "an interlocking chain unit configured such that chains reliably and firmly engage with each other and that the buckling and twisting of a rigidified chain portion are avoided." This is one example of several similar approaches to engineering tessellating geometries that mate via a particular motion to become locked and can only be released via the reverse motion. The current thesis extends the fabricable geometries from single beams to forking, angled and merging beams, sheets, curved surfaces and rectilinear forms.

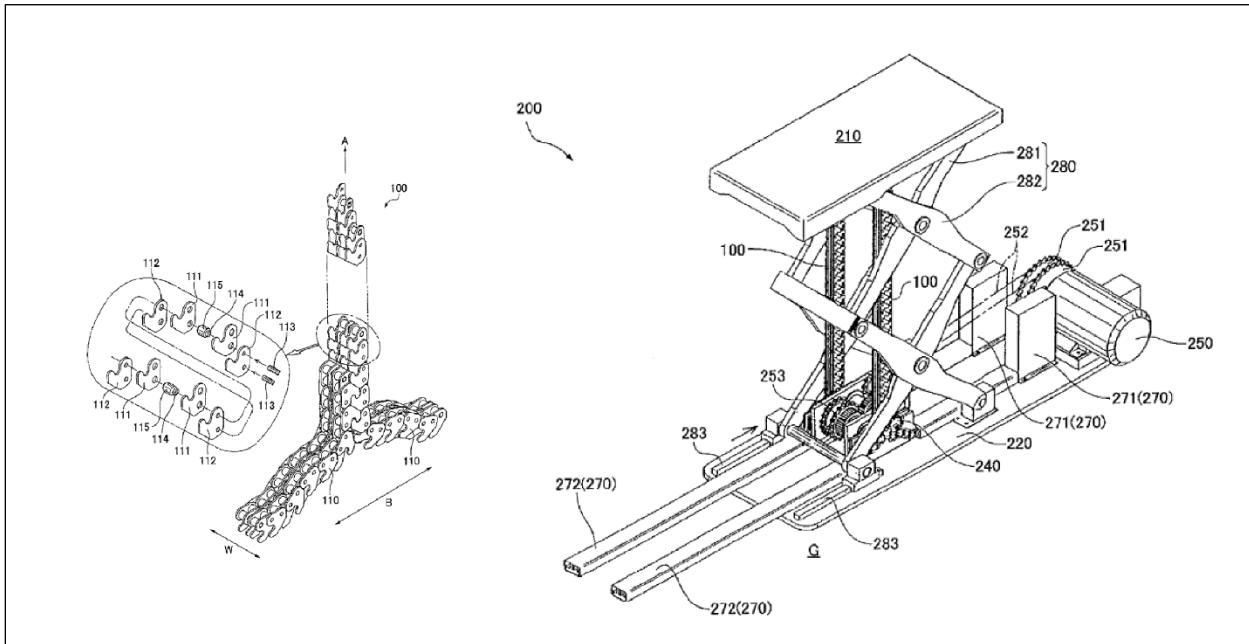


Figure 6 Zip chain actuator by Tsubaki, US Patent 9,482,313. Image credit: [23]

Recent work has gone into computational deconstruction of 3D surfaces into 2D shapes, including the Zippables project form the Interactive Geometry Lab at ETH where the edges of these 2D shapes are sewn to one side of a standard, modern manufactured zipper. When zipped together 3D geometry are reformed [24]. Another approach to zipped forms is presented by Kelly Delp, titled Zippergons [25], where 3D surfaces are decomposed into 2D shapes which contain rippling along their periphery. When the rippling is forced together the pieces are "zipped" into the form.

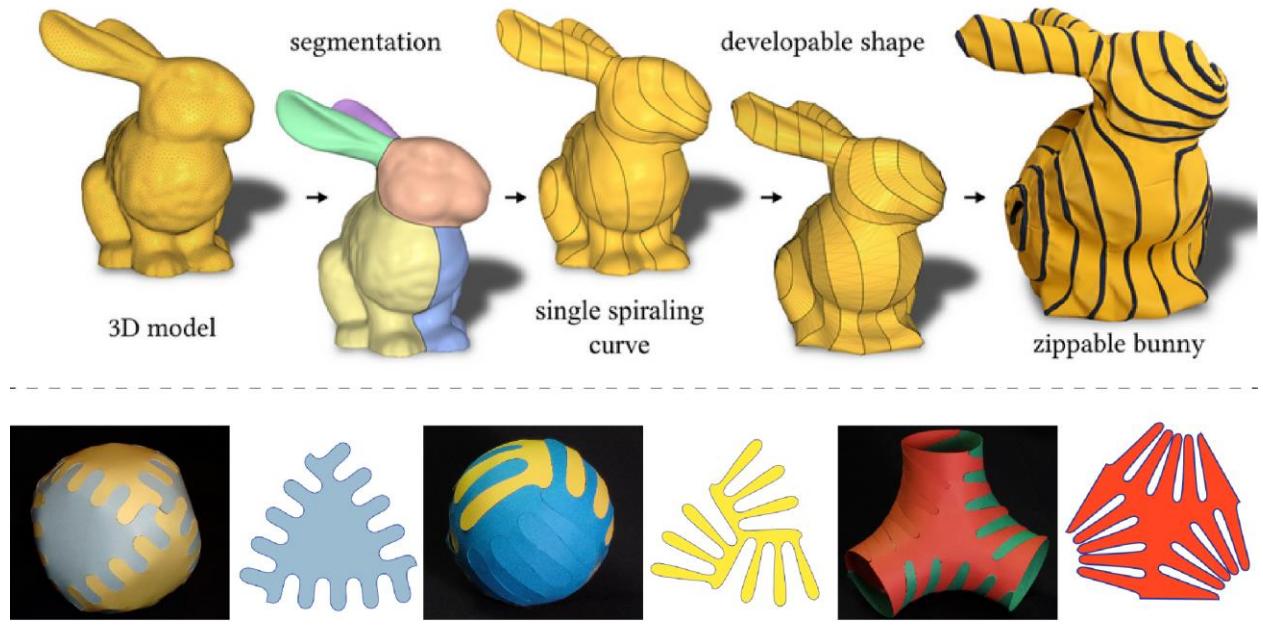


Figure 7 Diagrams from zipper related research project, Zippables and Zippergons. Image credit: [24], [25]

Robotic mechanisms that interact with zippers have also been explored by previous researchers. Shown below is a robotic zipper slider created by Adam Whiton of the MIT Media Lab Personal Robotics Group [26]. This robot, named Zipperbot, has a gear head which engages with the teeth on both sides of the conventional zipper. The geared DC motor drives the robotic slider forward and backward, zipping and unzipping the section.

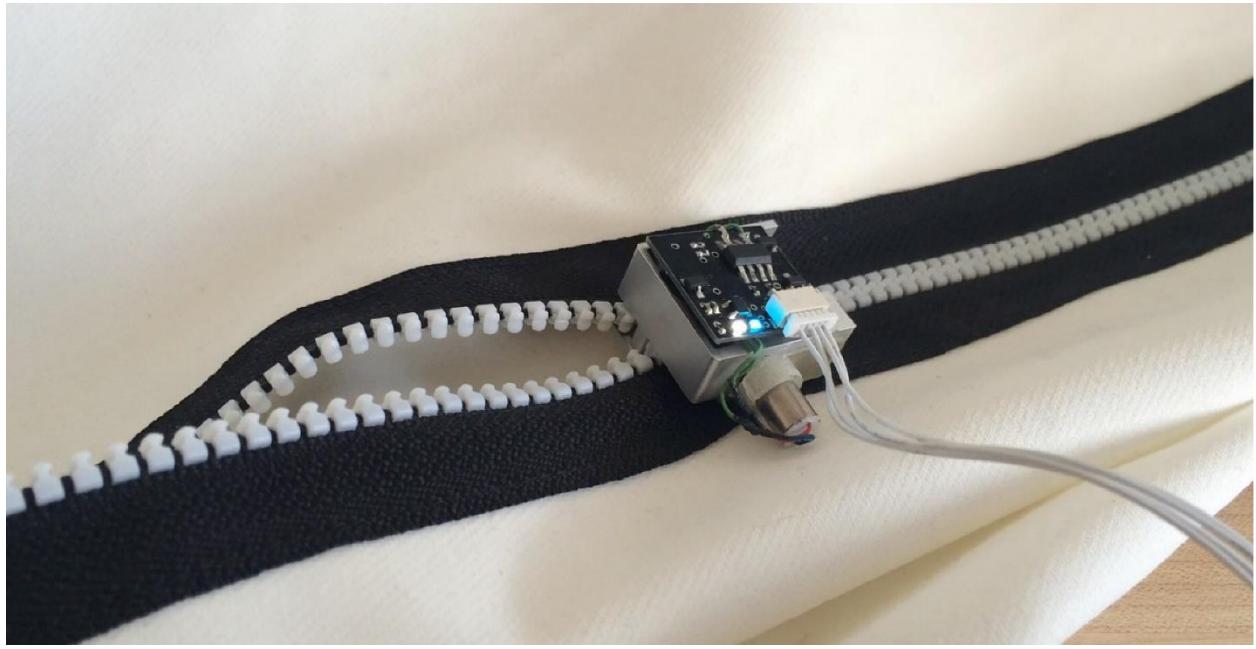


Figure 8 Zipperbot by Adam Whiton, MIT Personal Robotics Group. Image credit: [26]

## 2 ASSEMBLE, ZIP AND FOLD FABRICATION

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Inspired by biological fabrication, several parts were made to explore zipping and assembly-based fabrication. These prototype systems were considered on several metrics including ease of fabrication, ability to mechanize, projected method to span a gap, reusability and versatility of parts, novelty and intuition on strength to weight.

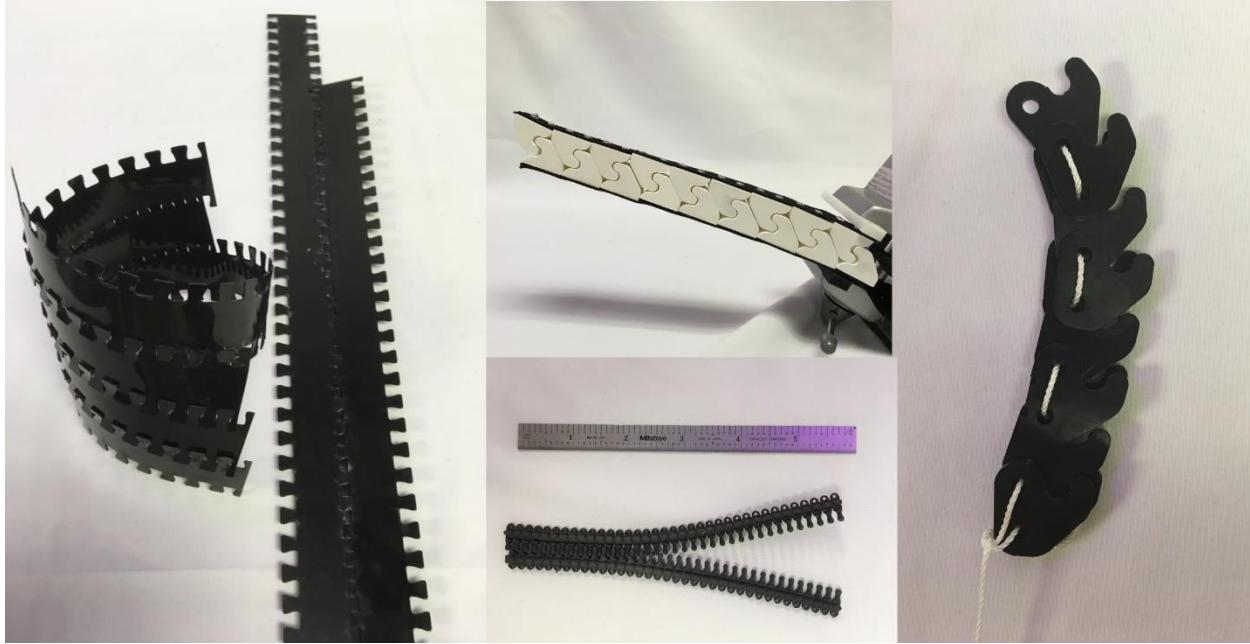


Figure 9 Prototyped parts for a zipper based fabrication system.

The fundamental geometry pictured below was found to be highly effective in locking two strands together, while maintaining flexibility when not mated. Additionally, it offers easy-to-fabricate discrete parts. The first few thousand prototype parts were made by cutting this geometry out of 3/16" Delrin sheets and pinning them into strands using push rivets. This is the fundamental geometry for the Zipped system.

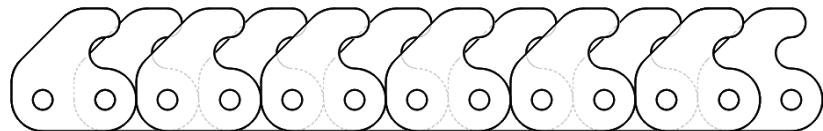


Figure 10 Core geometry used in several part types which locks into the Zipped system.

The Zipped approach is of interest for several reasons. Firstly, this system follows a hierarchy where individual parts can be considered 0D, they are assembled into 1D strands and those strands zip together and bend in a 2D space. Further assembly of the strands creates 3D structures. This creates a compelling mechanical analogy to biological molecular processes. This building system can be considered a digital material, where information can be stored and altered physically. The way parts were assembled from 0D to 1D affects what 1D structures can be zipped to form variations in 2D. The

2D topology affects what 3D forms are realizable. By templating a series of core elements various properties emerge.

The system proposed in this thesis is effectively a topologically interlocked material, though it differs from the traditional design as it does not require formwork. The assembly process also differs from traditional parts placement as Zipped pieces assemble into strands, then zip together to form structures. The assembled approach to construction also has many advantages as it allows re-use and re-purposing of the fundamental building block. The Zipped system has been described as "macro-scale Velcro", though the assembly and zipping require a controlled motion, versus the locking induced by contact from any direction as with Velcro. This allows the system to hold position without requiring any energy input and maintain strength until unlocked via the specific motion. The connection and disconnection does not damage the part in any way, thus leaving them unaltered for re-use. When considering global resource use, material life-time and manufacturing costs, designing for re-usability is imperative. This system allowing a diversity of fabrication to emerge from a small set of core parts and tools.

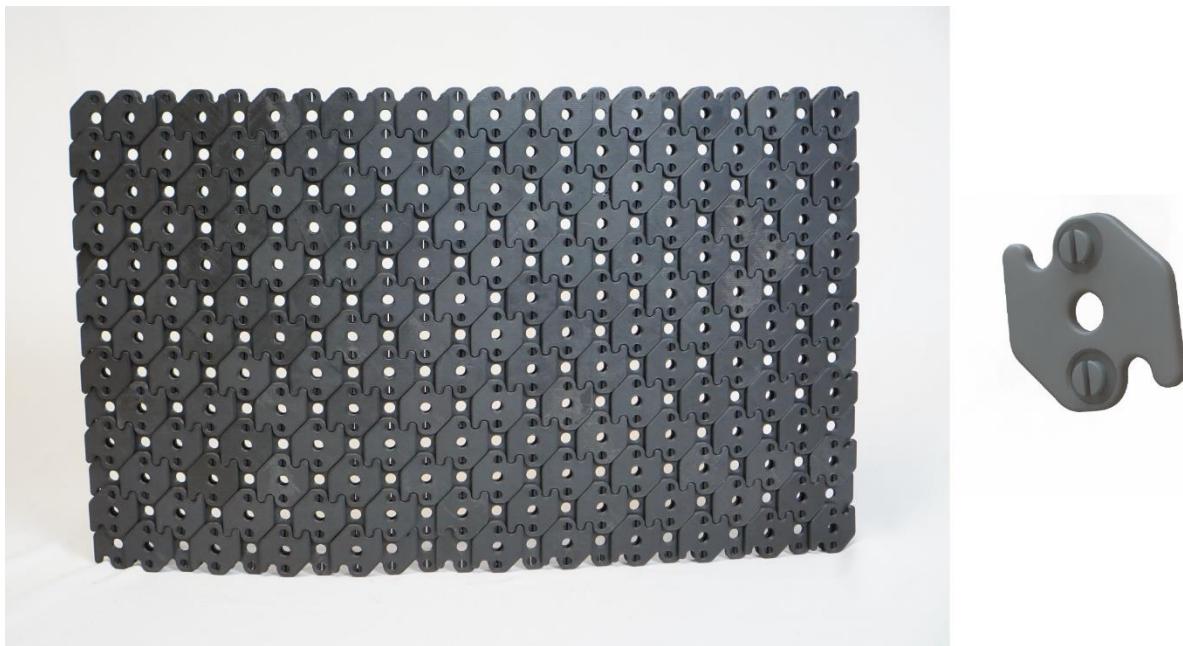


Figure 11 A flat sheet made from ZIPPED and a singular piece

Going from 1D to 2D, the mechanical mating of these strands is also of interest as the material properties of the zipped sheet versus un-zipped, or free, strand differ dramatically. In an unassembled state these strands are completely flexible, however when zipped together they form a rigid beam. The strength of these connections is repeatable and replicable as the elements are zipped and un-zipped. Strands can be assembled in several orientations which enable a variety of form as well as tunable, modular strength structures. The assembly of these strands exhibit isotropic material properties, which can be affected and altered via the geometry and connection mechanisms used to assemble the strands from teeth.

The repeatability of these strands, which allows robust mechanical locking between strands also enables an ease in automation of mobile robots that can locomote and localize on the structure relative to the building blocks [27]. These robots can be used to diagnose, interpret and sense the structures

environment. They also add fabrication capacity, allowing parts of the structure to grow, or be removed, enabling a structural-robotic system.



Figure 12 Picture from design review with the Army Research Lab where Zipped is introduced. May 1, 2019.

### 2.1.1 System element sizing

A study was performed to evaluate fabrication of this system with different sized parts. The picture below depicts prototype pieces fabricated at various scales using different manufacturing techniques and materials. Depending on the application, smaller or larger parts may be advantageous. Below parts are shown fabricated from bronze, cut on a millimeter scale with a laser micromachining system. Parts are made from Delrin at a centimeter scale cut on a CO<sub>2</sub> laser cutter. Parts are also cut on a CNC router from cardboard, which has the potential to increase multiple folds in size, up to the meter scale. Though the external geometry for each of these scales is identical, the revolute joint required to connect the teeth needs to be uniquely addressed for each system. For ease of manufacturing the meso scale parts were manufacture in bulk for lab experimentation.

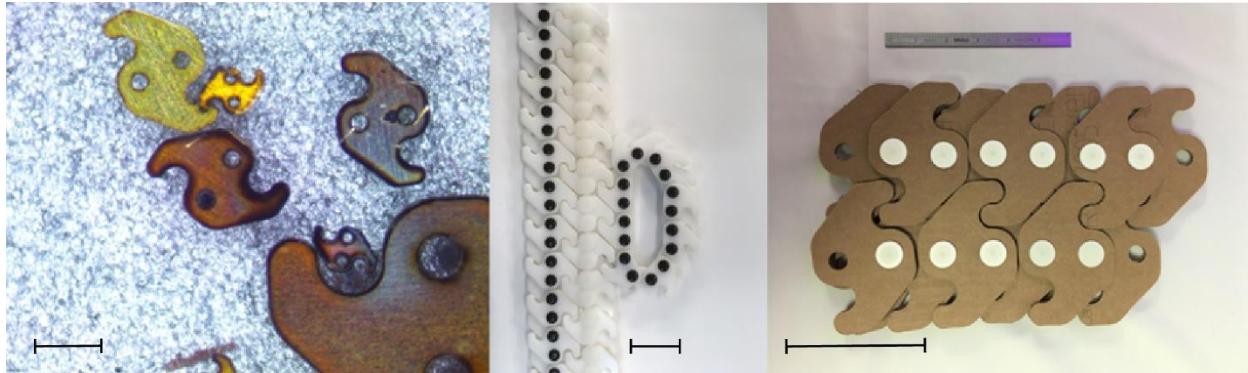


Figure 13 Scales of fabrication. The scale bars denote the width of the teeth: 1mm, 1.5in and 110mm.

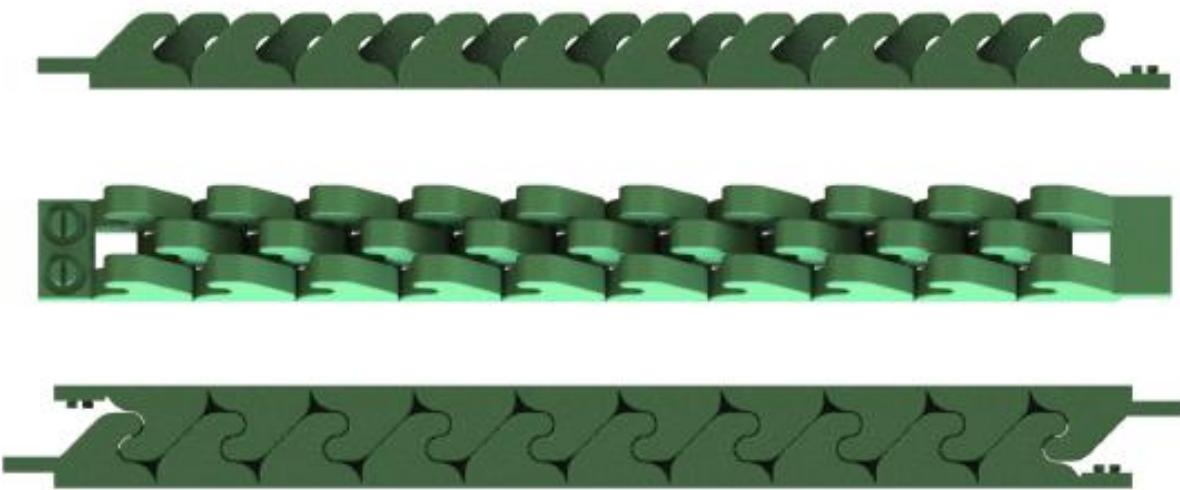
Below several inch-thick hollow sample were made to float allowing different configurations of rafts to be assembled. These pieces could be scaled much larger and rotomolded for large volume production. As large pins were required for these parts Aluminum rods were forged with a button end on one side and a collar was used on the other, a process which would also scale.



Figure 14 Large scale floating Zipped buoys

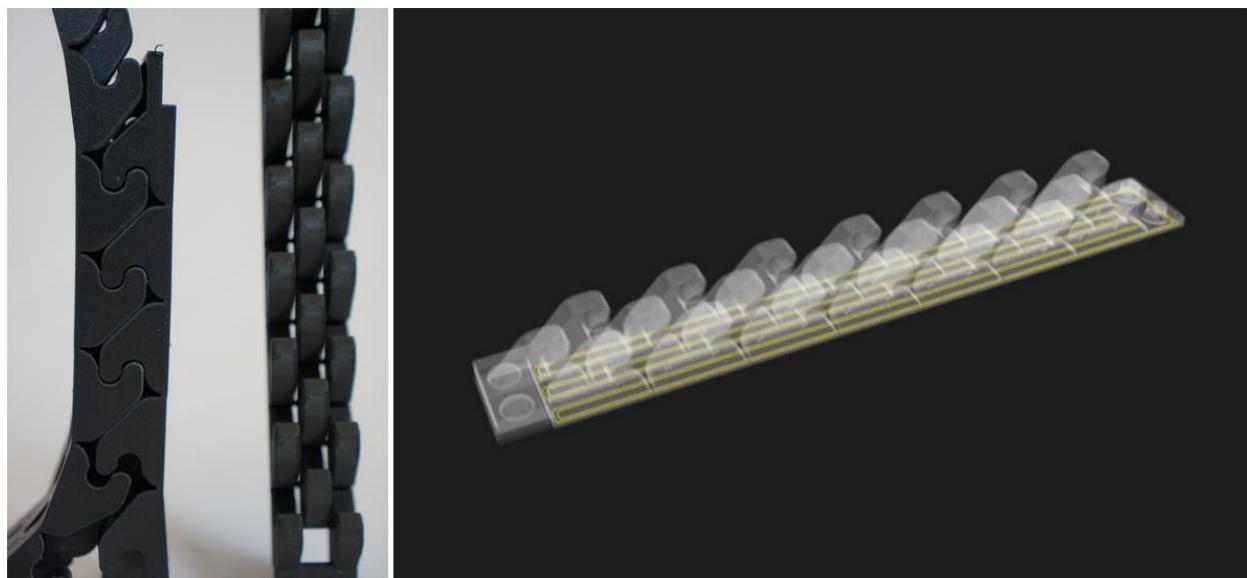
### 2.1.2 Monolithic, Multi-Teeth Strands

It is possible to produce strands which are fabricated as a single unit. The connection between the teeth can be made either from a thin connector of the bulk material, a living hinge, or by insert molding a reinforcement such as fibers. Samples were made with several layers of teeth that allow strands to be offset enabling extension in the X and Y direction. Though it is more challenging to manufacture, it was also shown that double sided monolithic strands would function similarly to double sided strands assembled from double sided teeth. The advantage with this monolithic approach is the ability to increase the strength between the teeth by inserting and adjusting a fiber reinforced spine that runs lengthwise down the strand.



*Figure 15 Monolithic Zipped strands.*

The samples pictured below are printed on a Mark Two printer which allows continuous lengths of fiber to be inlaid. In this case Kevlar is used because it is flexible and allows the strands to bend and lock. The image below on the left shows an internal view of the part, depicting where the fiber is placed with yellow. This design is particularly hard to manufacture at scale as there are multi-directional overhangs which prevent molding it as a single piece. Other manufacturing processes could be developed to fabricate this design however, including molding each tooth independently and laminating them to a substrate spine with an adhesive.



*Figure 16 Monolithic strands. (L) a picture of manufactured monolithic strands (R) internal view showing Kevlar reinforcements*

## 2.2 DIGITAL MATERIAL FUNCTIONALITY

Digital materials allow additional functionality to be embedded into the building blocks. For instance, teeth could be a mix of resistive and insulating elements to route power and data through a structure. Differences in the teeth patterning along a strand could act as a barcode to embed information and flag certain areas of the structure. For instance, starting to zip an addition onto a structure or stopping that assembly can be signaled via three white teeth in a row.

In addition to physical signals, digital signals can be imbedded in parts of the structure and interpreted by robotic carriages which ride along these structures. The pieces could have varying physical or optical properties, affecting the structures performance or appearance. Structural sensing is useful for diagnostic purposes if the structure is likely to be damaged, it also can allow the robot to localize on the structure by determining where it is from the number of parts they have passed. Each strand could have a conductive line so that power can be fed from one tooth to the next to allow a continuous current through a beam acting as a tether for robots which locomote on the structure.

Incorporated conductive lines were prototyped via electroplating, conductive adhesive tape attached, heat pressed inserts as well as multi-layer stacking of conductive and insulative parts. This frees the actuated elements from requiring individual batteries, which is a common concern for multi-actuated systems. Shown below is a demonstration where a powered fan turned a miniature wind turbine on top of a zipped beam with conducting lines. The power produced was measured at the bottom of the structure.

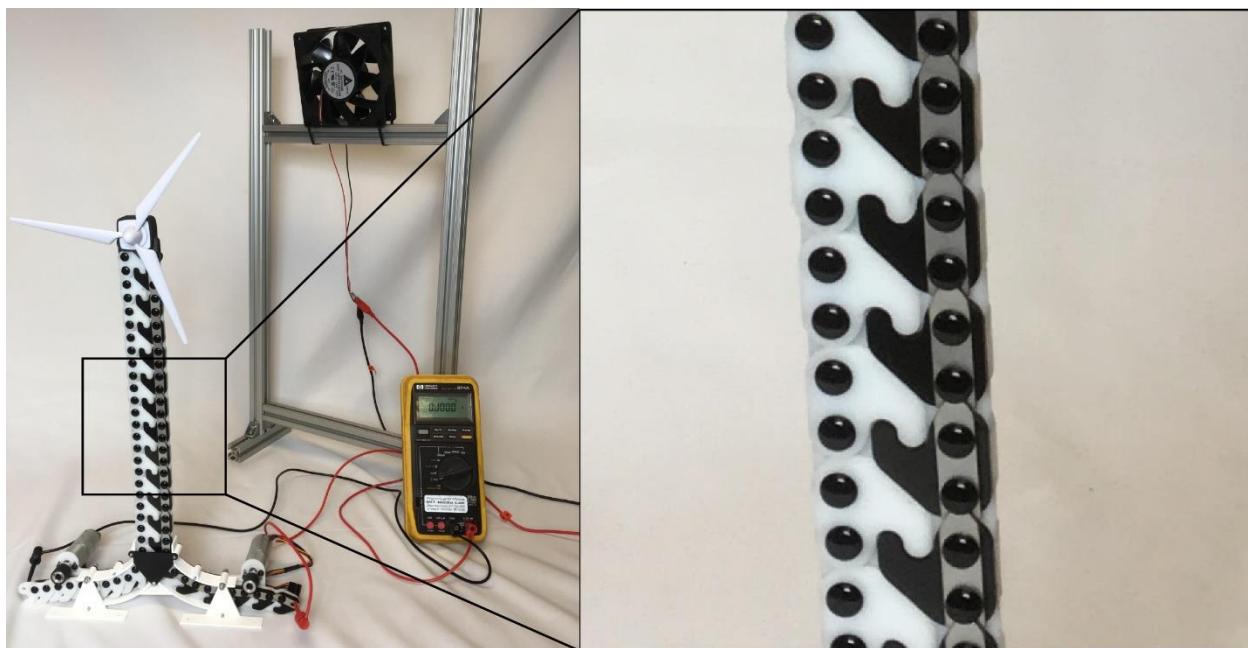


Figure 17 Demonstration of a stand with conductive power lines

### 3 THE ZIPPED SYSTEM

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The Zipped system is structural-robotic approach towards architectural scale assembly of digital materials. Analogous to many biological systems the Zipped system relies on hierarchical design rules to assemble larger, more complex structures. Design decisions when assembling lower level elements affect fabricability of higher-level primitives, in this case the tooth selection determines strand functionality. Strand design affects what 3D structures can be created from it.

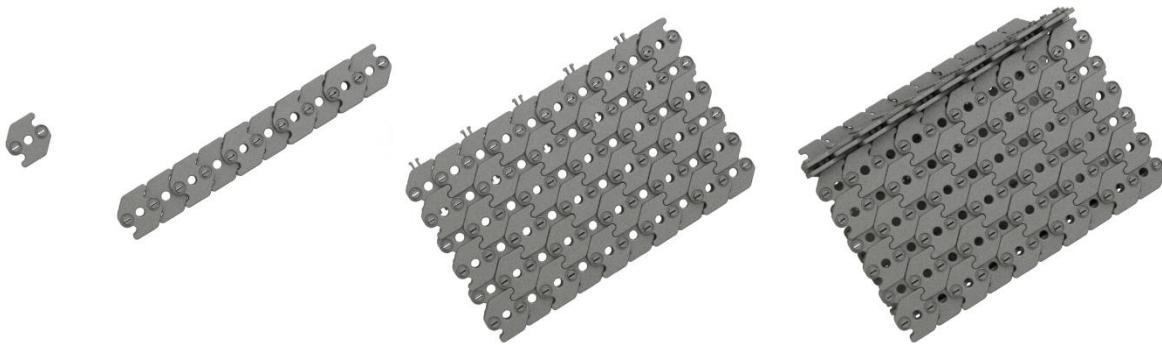


Figure 18 0-Dimensional part assembled into a 1D chain, then a 2D sheet, then a 3D channel.

The Zipped system is assembled from fundamental building blocks referred to as "teeth". Each tooth has two pin joints connecting it to another tooth. The teeth are offset by these pivot points and stacked in layers making a linear assemblage referred to as a "strand". Strands lock together by zipping the teeth, as the teeth have a mating geometry. No force is required to maintain the structural connection, the strands can only be unlocked by pulling one end of a loose strand backwards along the length of the strand. To describe and evaluate the system the following coordinate system was defined and is used throughout this document.

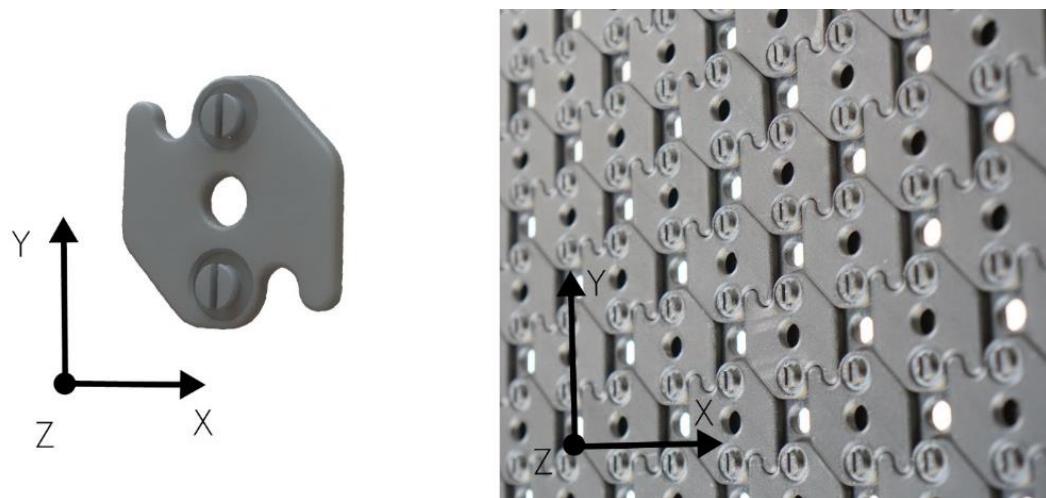


Figure 19 Coordinate system used to describe Zipped.

The stages of assembled hierarchy listed and pictured below. These elements are assembled hierachal to form different functionalities in assemblages. I.E. tooth selection effects the functionality of the assembled strand, which then alters which structural primitives can be fabricated.

**Teeth [0D]** Single sided, double sided, angled, chiral angled, flexural, orthogonal single and double sided.

**Strands [1D]** Assembly of these core components form strands, which can be 2+ teeth in layer thickness. Due to the geometry of some of the teeth parts edges on strands differ, for instance, the edge coming off a 90 deg part only allows a single toothed spine.

**Structural primitives [2D]** Beam, sheet, bend, loop, forked branched beams, merging beams

**Structures [3D]** Connected structural primitives

**Mechanisms [4D]** Actuators combine with structural primitives to form mechanisms and robots.

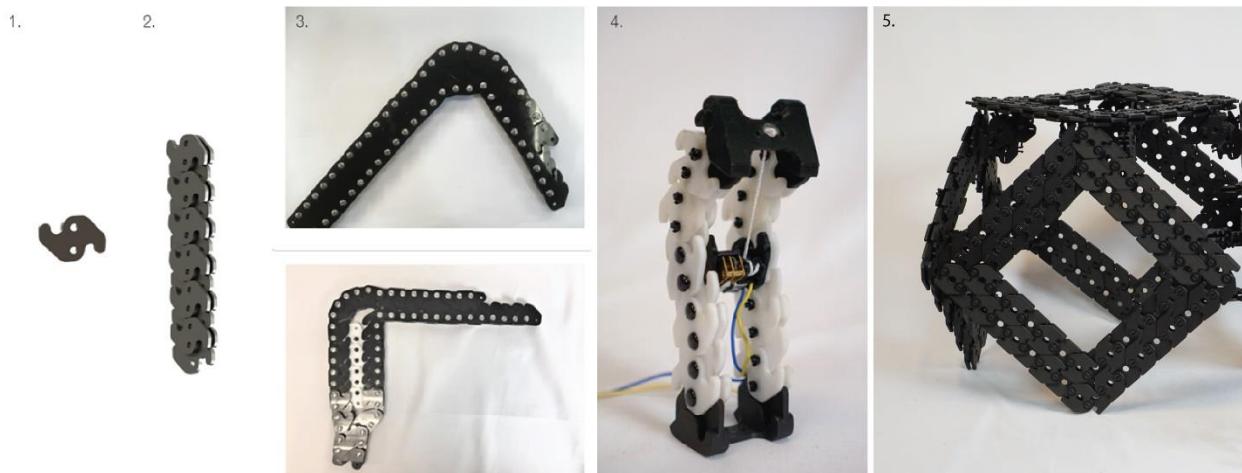


Figure 20 1. tooth, 2. a strand, 3. sample structural primitives, 4. an actuated element or mechanism, 5. a prototype structure

Note: Throughout this document the term parts, pieces and teeth are used interchangeably. Teeth are referred to as being manufactured or fabricated, whereas structures created from those parts are referred to as assembled or constructed.

### 3.1 OD PART DESIGN

The teeth are designed to allow a flexible chain to lock into a rigid beam. They are a tessellating geometry designed to fill space for maximal compressive strength, where the back side of the tooth acts as a strike plate with its interlocking neighbor. The tip of the teeth are nested to stop shearing between two strands. Experiments were performed cutting teeth of various extensions to determine the optimal tooth design considering the trade-off between the shear force needed to pull strands apart which increased with tooth length and ease of zipping which decreases. This tooth geometry was designed into many parts which are shown below. The single tooth part type can only zip linearly, but also can be used to make a clean, flat perimeter. The double-sided toothed parts rotates the tooth around itself which enabled flat sheets to be created. These parts that are more space filling, but do not allow orthogonal snapping on Zipped strands. The larger iteration allowed strands to mate and branch orthogonally in plane, thus it is referred to as an orthogonal part. The orthogonal part is shown with added side snaps that allows mating of orthogonal planes by snapping the figures on the edges into another parts central hole. Also pictured are a base and stand-off spacers.

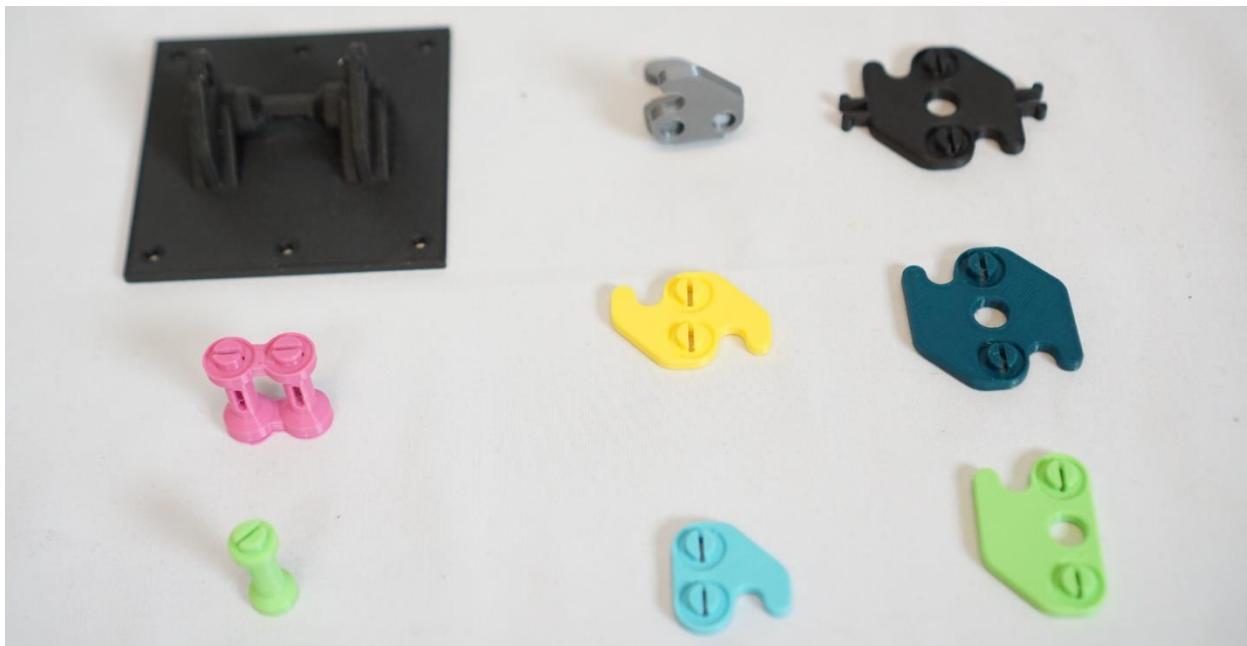


Figure 21 3D printed parts with snap fit joints to connect the pieces

Due to ease of fabrication and in-house scalability these parts, material system and size, were selected. The following two parts were selected as minimum part types and used for most of the physical demos pictured in this thesis.

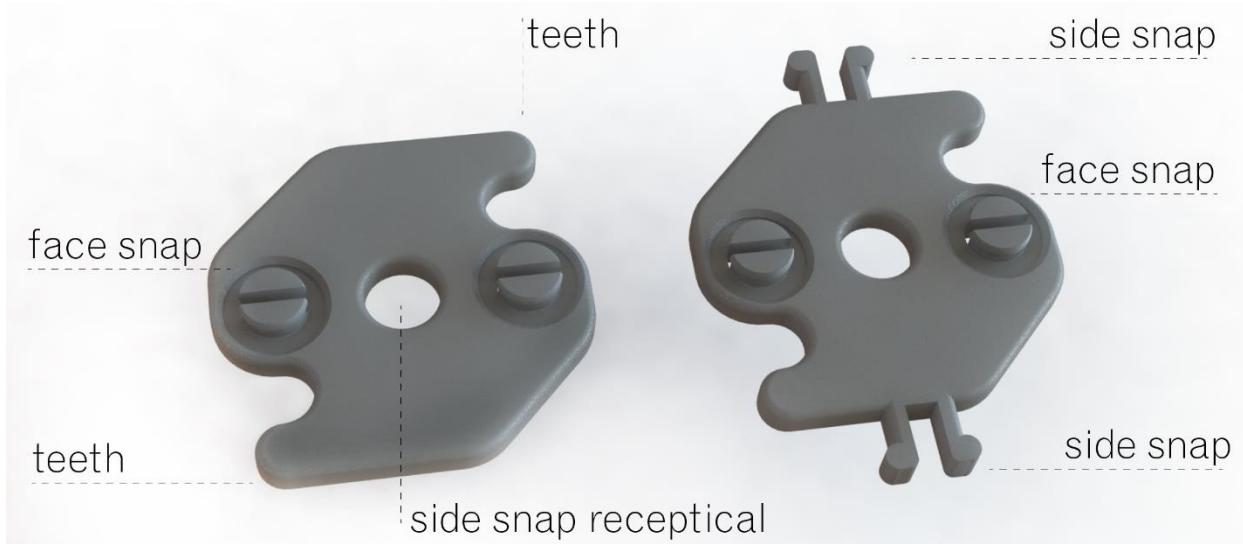


Figure 22 Orthogonal parts with labeled features. The positive side containing face snaps is shown, the negative side, not shown, has the mating cavity.

Though the orthogonal pieces are less space filling than the double-sided part designs, they allow a higher range of structural primitives, as the parts can mate orthogonally on a flat plane. This is possible because the snaps are the same distance from each other on a single part, as they are from snaps on their neighboring, adjacent part. A flat sheet of Zipped strands made from the above design has regularly spaced pins in the X and Y direction. When one of these pieces is mated orthogonally to the direction of the strands it can be referred to as a “staple” since it ties and locks the two strands together. Though strands made from orthogonal parts cannot zip with strands made from double-sided parts, they can fit together in the same construction using the face snaps, as shown below.

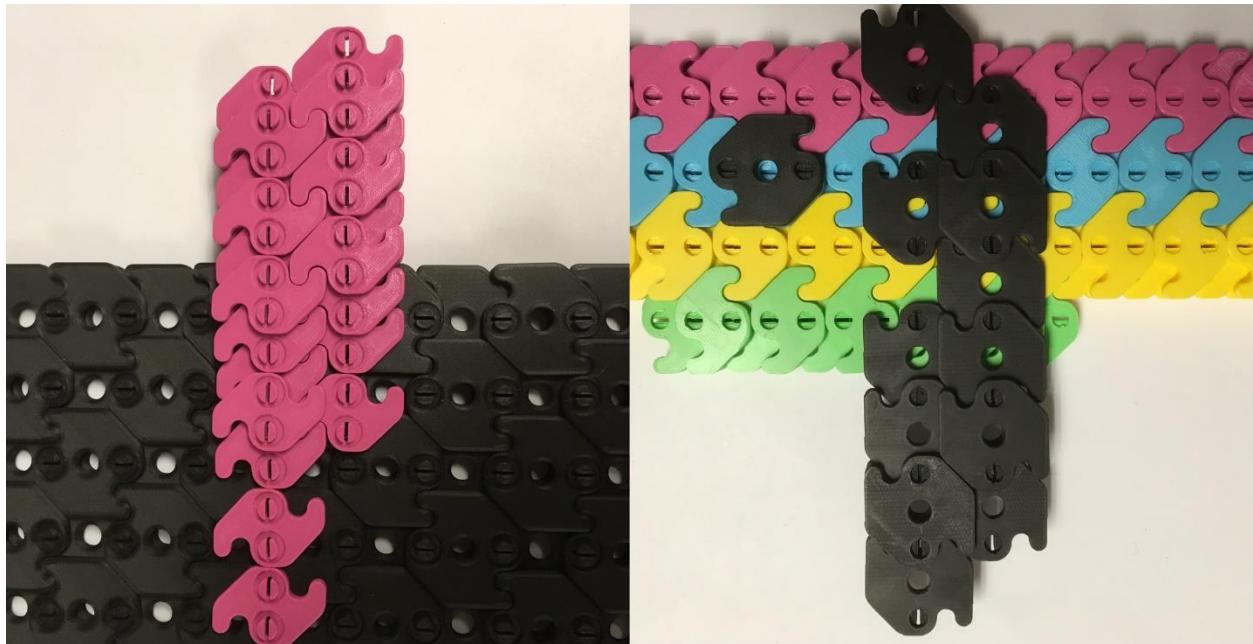


Figure 23 Combination of orthogonal staple parts and double-sided parts, the original tooth geometry.

## 3.2 GOING FROM 0D TO 1D

Teeth connect to each other along an axis to form a strand. Un-locked, or un-zipped strands are flexible. Their motion is constrained to the YX plane, but they can bend and curl freely in that plane enabling their use as chains and flexures. A strand needs to be three teeth thick to fully constrain another strand when zipped. A strand many teeth thick can be referred to as a “stack”. The connection between the 0D teeth parts needs to be evaluated independently for each tooth design and scale, as various features and manufacturing methods are applicable at different scales. Trade-offs are discussed per connection type: press-fit, pin rivets and snap fits.



Figure 24 Chain of zipped elements, not constrained and thus flexible in plane.

### 3.2.1 Press fits

Though press fits are molded into a variety of toys allowing them to connect it is not an ideal design for structural plastic applications as the plastic will deform over time, changing the strength of the connection. Nor are press-fits ideal for metal parts as metals have a high yield strength which prevents the press fit from rotating in their current geometry. Rotation around the joint is needed when strands zip and unzip. This rotational motion can also lead the press-fit to “walking off” as a press fit connection uses the same frictional force to hold the pieces together as fights their rotational motion. To calculate the axial load supported by the press fit the following equation can be used, where P is the pressure between the press fit materials.

$$F_{axial} = P \cdot \mu \cdot \pi \cdot D \cdot L$$

This can be compared to the torque resistance, caused by the same friction:

$$2 \cdot T = P \cdot \mu \cdot \pi \cdot D^2 \cdot L$$

Since the connection is caused fundamentally by the same force these equations are not independent, and thus there is no way to optimize the design. At the point where the frictional force between the peg and the hole exceeds the applied torsional load the press fit will no longer be able to rotate.

Note: It is possible to use a pin and design an interference fit for the two exterior pieces on a strand while keeping the interior pieces looser. This would hold the teeth in place and allow the strand to rotate, though it would necessitate different parts for the sandwiched and exterior pieces.

### 3.2.2 Pin rivets

A pin may be used to join teeth. This was prototyped using a variety of push rivets, two-part rivets, collet pins and fasteners. Axial pre-load on the pins or fasteners is to be avoided as it linearly increases the friction between the face of the teeth, increasing the force it takes for them to rotate on each other.

$$\text{Frictional force} = \mu * \text{normal force}$$

The friction between teeth parts also reduces the shear force experienced by the pin, which was often seen to be the point of failure, thus it is a trade-off. To increase load capacity the pin can be redesigned to be larger or made of a stronger material to increase its strength, however, the tensile load on the hoop of material created by the hole for the pin must also support this force,  $F$ . The smaller result from following two equations indicates where failure will occur, either the hoop in tooth or the pin.

$$F_{\text{pin}} = \text{Pin material shear yield} \cdot \pi \cdot R1^2$$

$$F_{\text{tooth}} = 2 \cdot T \cdot (R2 - R1) \cdot \text{Tooth material}$$

Polarized light was used to capture the internal stresses of the teeth via photoelasticity in the current design [28]. Stress concentrations peak in the loop around the pin as well as the contact between the tip of the teeth. The maximal load measured by the Instron during this experiment was  $\sim 200\text{N}$ .

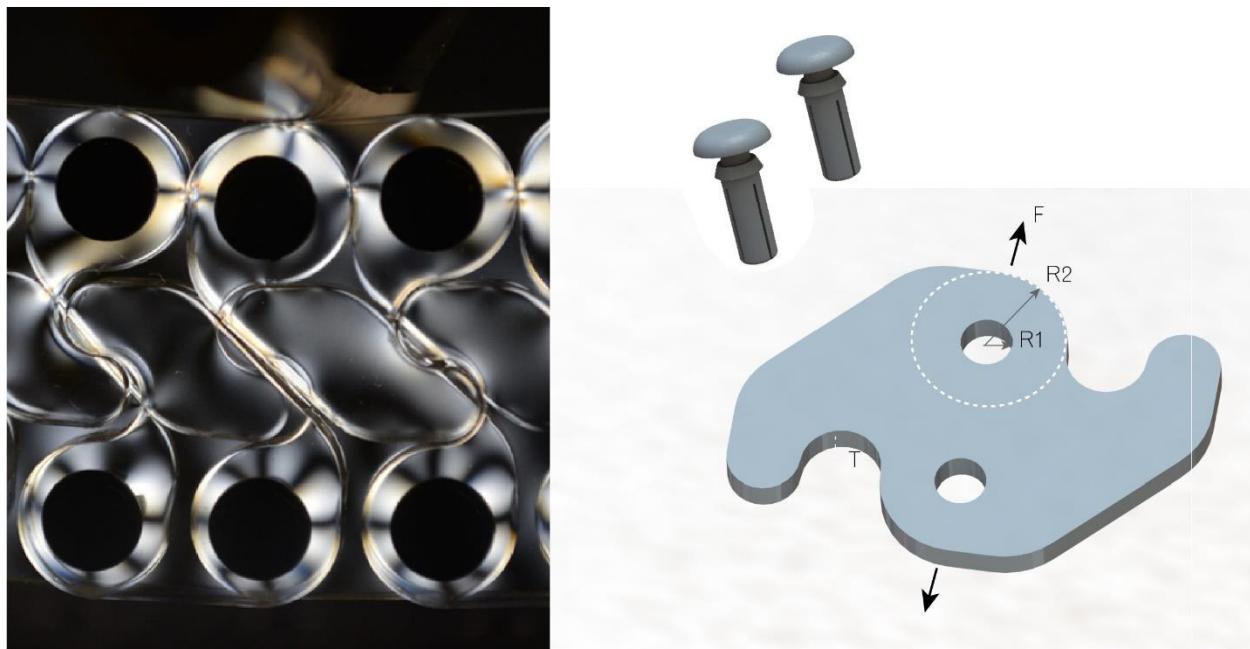


Figure 25 Photoelasticity experiment showing stress concentrations in the hoop. Push-In rivet and force diagram for 2D teeth

The major disadvantage of pinning is the limitation on number of parts stackable. The pin will be a specific, predefined length and can only accommodate a certain number of teeth stacked together. For instance, if a pin is 0.35" long and the teeth are 0.10" thick, only three of them may be stacked, and if fewer are stacked there will be significant excess play between the parts. It also prevents additional parts from being stacked on top as the pin would need to be removed and something would need to hold the parts together bracing them while a new pin is inserted.

### 3.2.3 Snap fit

Because plastics deform under prolonged deformation, creep, it is important to design these parts such that they are not experiencing a constant load [29]. For this reason, press fits are not a good solution to mate plastic parts, as the mating force between the press fit parts will change with time. Therefor snap fits are considered best practice as a cost-effective, moldable feature to mate plastic parts. These snap fits are designed to flex under a mating force while being inserted and removed. It is also possible to design irreversible snap fits, which would not allow the parts to be removed, like a zip-tie.

For the zippers the connection between the teeth is also a rotational axis they need to pivot around, therefor an annular snap fit is required. An initial design intended for 3D printing is shown below. The feature size is limited to 0.075" due to the limitations if the FDM machine.

When the snap fit is pulled axially, as in the figure below, force is translated inward to deflect it. The strength of the snap fit is determined by the force needed to deflect the snap inward. For a cantilevered beam, which is effectively what the snap fit is, the deflection is found to be a function of the following:

$$\delta = \frac{F \cdot x^2}{3 \cdot E \cdot I}$$

Below are pictures of the snap fit design as well as a force diagram Where F is the force needed to pull the snap fit off by the resulting vector,  $F \cdot \sin\theta\cos\theta$ , the amount of force needed to deform the cantilevered beam.

Over molding can allow snaps of different materials to be molded into the part. It is possible to leverage common place sewing snap for this design, which are stamped from sheet metal and can be bought affordably in bulk.

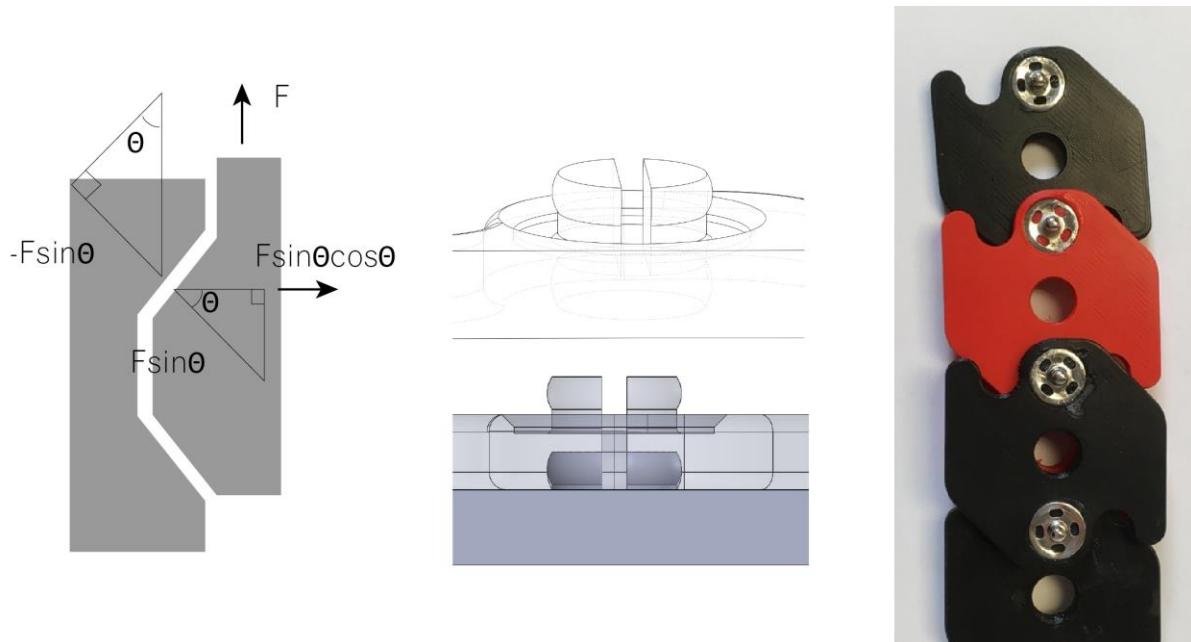
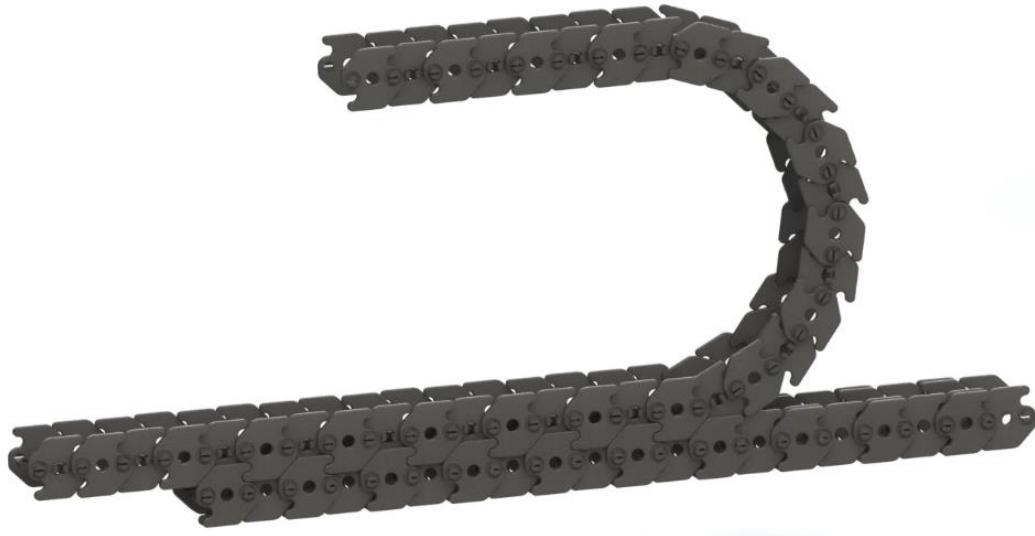


Figure 26 Snap fit geometry force diagram, same material snap fit design and a prototype using sewing snaps.

### 3.3 GOING FROM 1D TO 2D

Strands interlock via zipping and geometrically constrain the parts. For strands to interlock however they need to be symmetrically mirrored, thus allowing them to fit together. Strands have a directionality in the orientation their teeth point. To connect strands together the teeth need to be in the inverse direction. Strands contain (at least) two edges, with a flexural spine along the central axis. The strands are planar linkages, thus as the strand unfolds onto the edge, this motion is confined to the plane defined by the edge.



*Figure 27 Teeth directionality in a Zipped plane shows the front strands ability to lock to the bottom strand.*

Several pieces require mating geometry that not all other pieces contain. For instance, the side snaps need a hole, and all leading edges require strands of specific types to mate. The teeth need to mate for two strands to lock, therefor the leading-edge teeth need to be in opposite directions on the same plane. If an angled edge is assembled onto part of the structure, the strands that follow need to be assembled on that angle, which prevents additional construction in that plane.

Some base structural primitives are shown below using 2D manufactured parts with plastic rivet connectors. This figure demonstrating the system's capacity to lock to itself, create joints, form angles and mend local breaks. Strands with non-mating teeth will not zip. By orienting strands with a segment of mating teeth, then non-mating teeth for another segment, bends and branches can be formed. This method was used to create various structural shown in the bottom row of images in Figure 28.



Figure 28 Structural primitives. On the top row strands are shown locking to themselves. The middle row shows a break in a strand and connector strands. The bottom row shows controlled angles.

The in-molded pin design, where one side of the tooth is positive and the other negative means that stacking pieces will always be in the correct orientation. Connecting teeth with rivets or external pins allows strands to be assembled with teeth pointing in opposite directions along the same strand creating stops where the strands cannot zip.

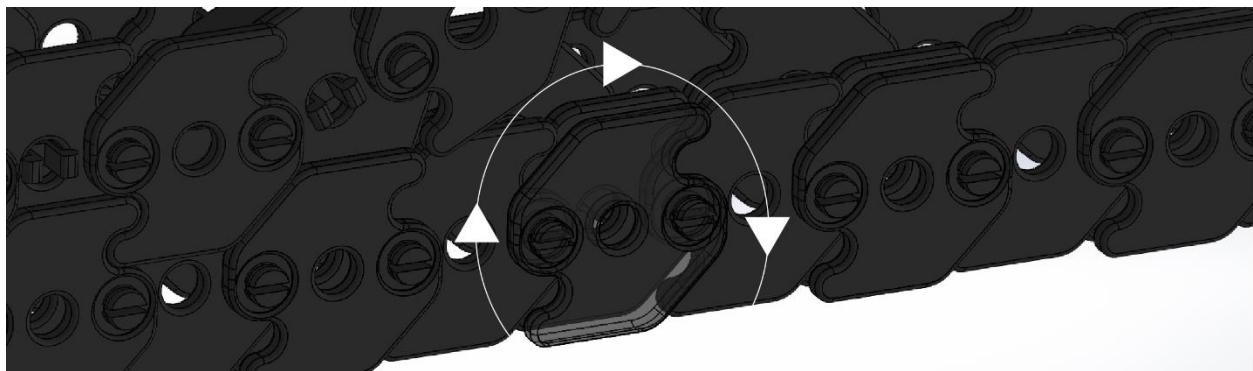
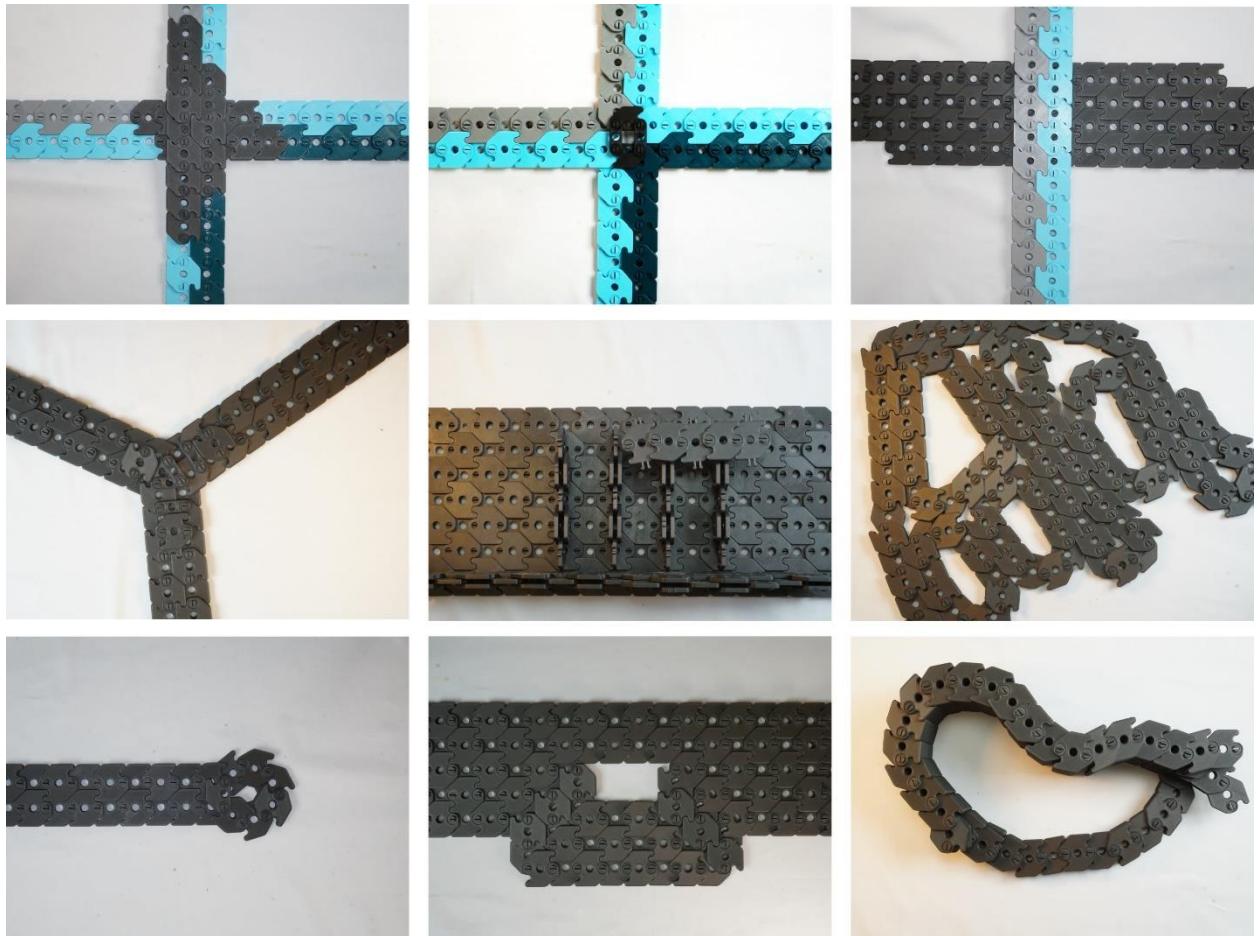


Figure 29 The part is auto-oriented because of the snap fit teeth.



*Figure 30 Structural primitives with snap pieces demonstrating their ability to snap orthogonally or along the zipping vector. The middle image shows side snaps constructing vertical walls. This design allows Zipped to branch and merge in the same way as the 2-dimensionally fabricated pieces shown previously, however the snap allows these pieces can stack indefinitely, enabling construction of high, flexible walls, as shown in the bottom left.*

### 3.4 3D DESIGN

Snaps can be added to the edge of the wide, orthogonal mating teeth. The snaps on the side of the part fit into the holes on the mating structure allowing a repetitive connection the length of the strands. These side snaps allow the parts to lock onto each other to connect orthogonal planes.

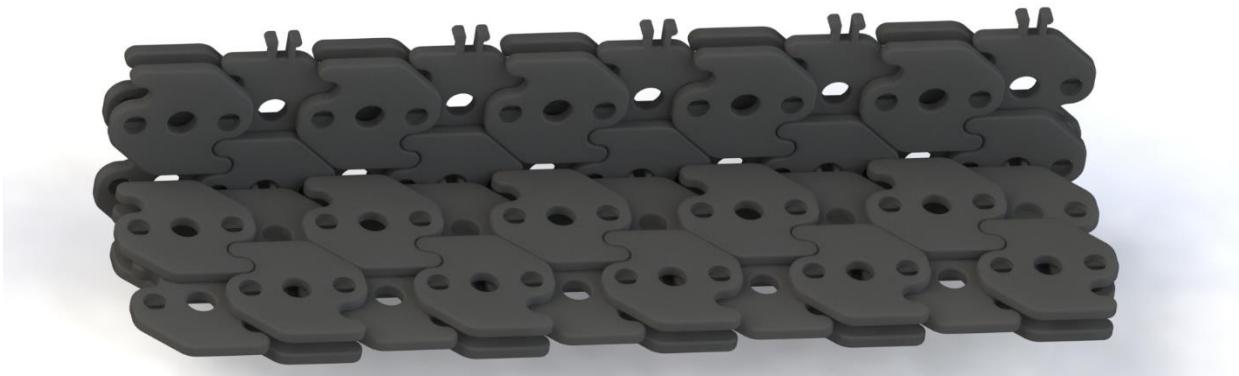


Figure 31 Side snaps connecting two sheets

In addition to allowing a wider range of structures to be fabricated, side snaps creating T joints can be used to increase structural performance as shown below. Here 28" sections with different configurations are compared. A single strand, two strands, a strand connected via side snaps to another strand which is oriented vertically in a sideways T configuration, and a strand connected via side snaps to another strand oriented horizontally in a T configuration. This shows the only self-supporting section of this length is the vertical T configuration.

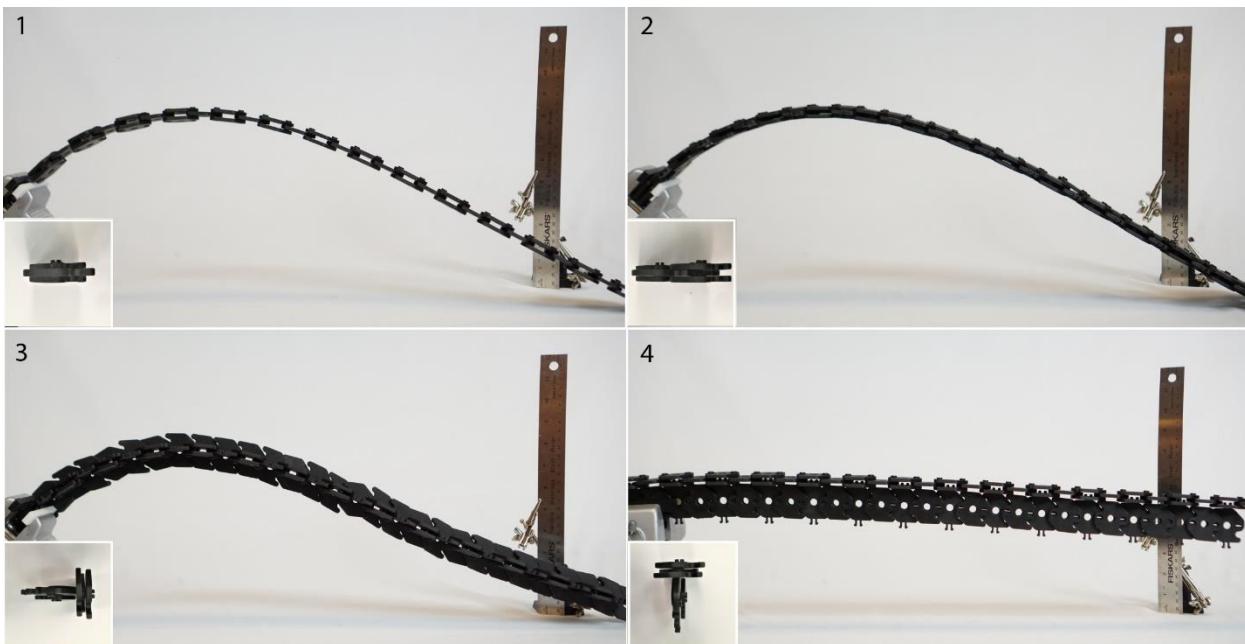


Figure 32 Cantilevered beam test on 28" sections. Profiles of each is displayed in the lower left and a 12" ruler is propped vertically on the right of each image.

The ability to construct flat, semi-flexible beams adds to the design space. For instance, three strands are used below to make a roadway for toy cars which can connect multiple levels.

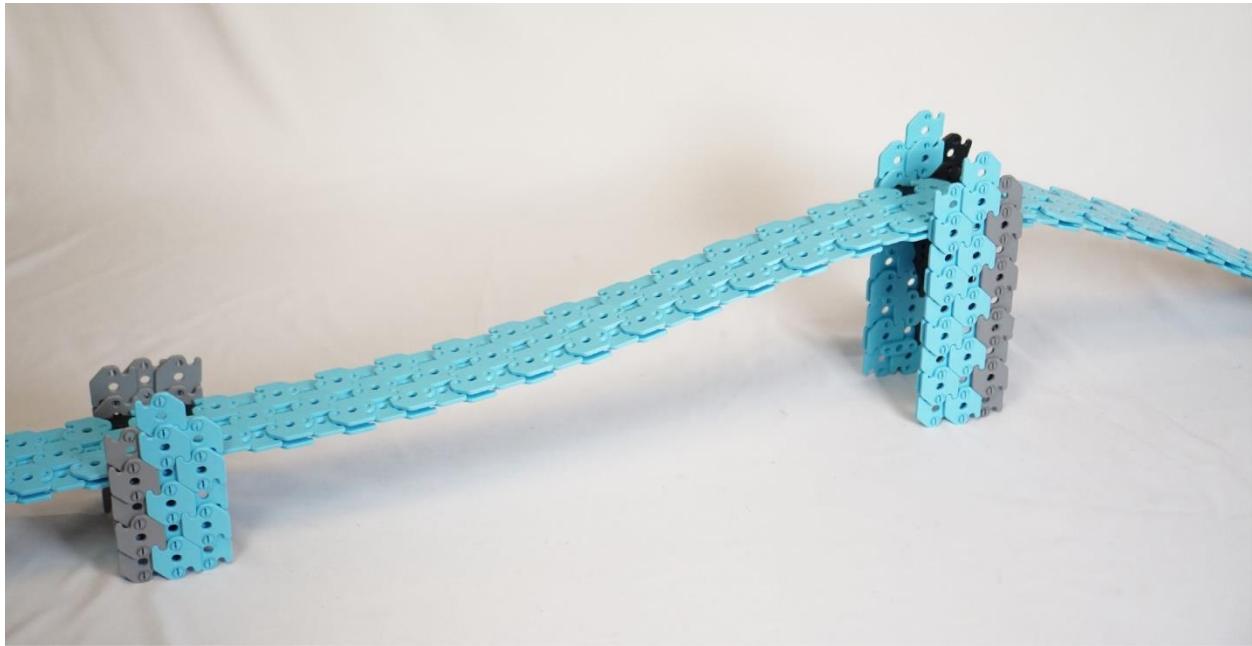


Figure 33 Track for toy cars which utilizes the flat zipped strands' flexibility to bend and create adjustable height ramps.

To reinforce beams orthogonal strands can be mated. Strands with side snaps can be attached in the X or Y direction on the negative side of the parts, or along the X direction on the positive side as the face snaps interfere in the Y direction. Pieces with side snaps may also be used within a sheet however they limit the locations at which other parts can side snap due to the interference shown below.

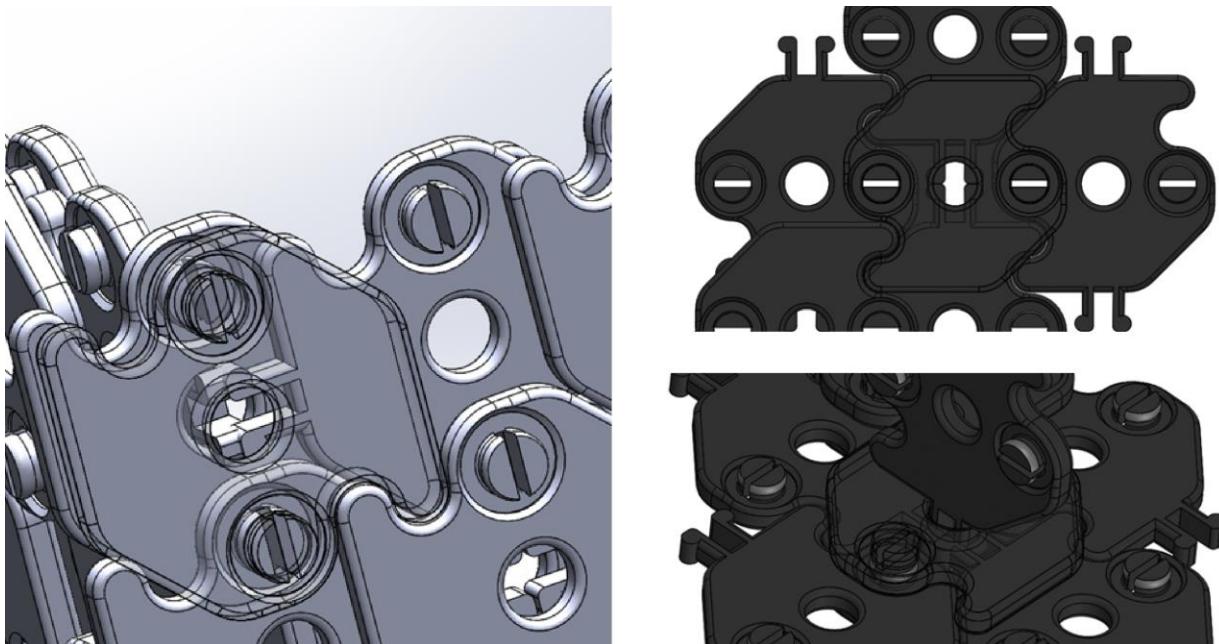


Figure 34 CAD showing side snap placement and interference. If parts that have side snaps are embedded in a sheet their side snaps will block the holes used for mating other side snaps.

If only a single side snap is present it can act as a revolute joint. The rotation is limited on the positive side however such that the part does not interfere with the protruding face snaps as shown below. This interference also prevents side snap strands from locking along the Y axis on the positive side of a sheet.

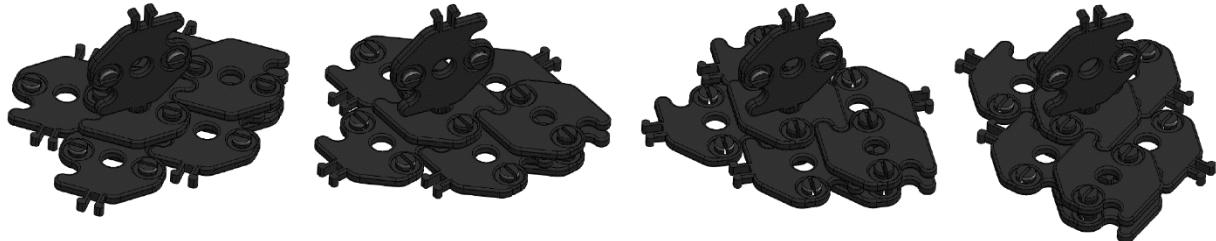


Figure 35 Single side snap connected to the face of a sheet which can be used as an out of plane revolute joint.

### 3.4.1 Angled teeth



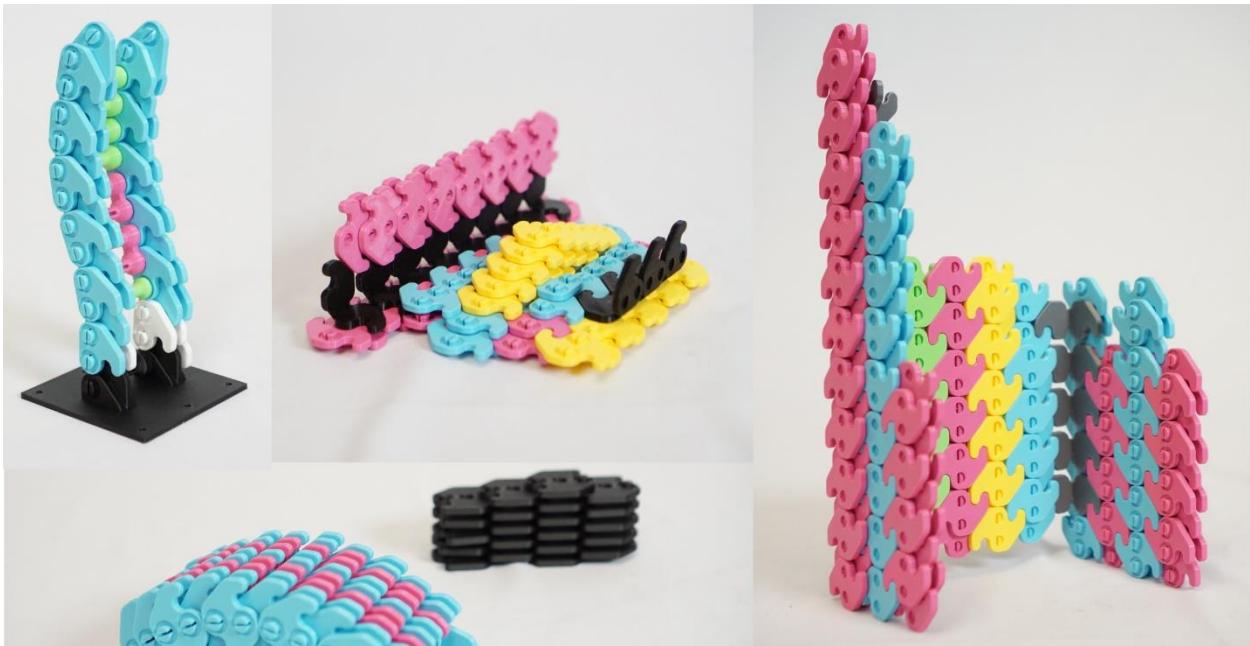
Figure 36 Variety of teeth, angled and flat, and strands assembled from them.

Parts can be manufactured with two teeth orthogonally connected forming a rigid angle. This allows strands to zip in orthogonal planes. Pictured above is a variety of angled and flat teeth and strands assembled using them. Due to geometric limitations it is only possible to make a single row of teeth protrude from an angled strand, and non-angled pieces are required to form the backer for angled pieces. The strand containing the angled pieces can only bend in a single plane which is defined by non-angled pieces thus the zipping order is defined.



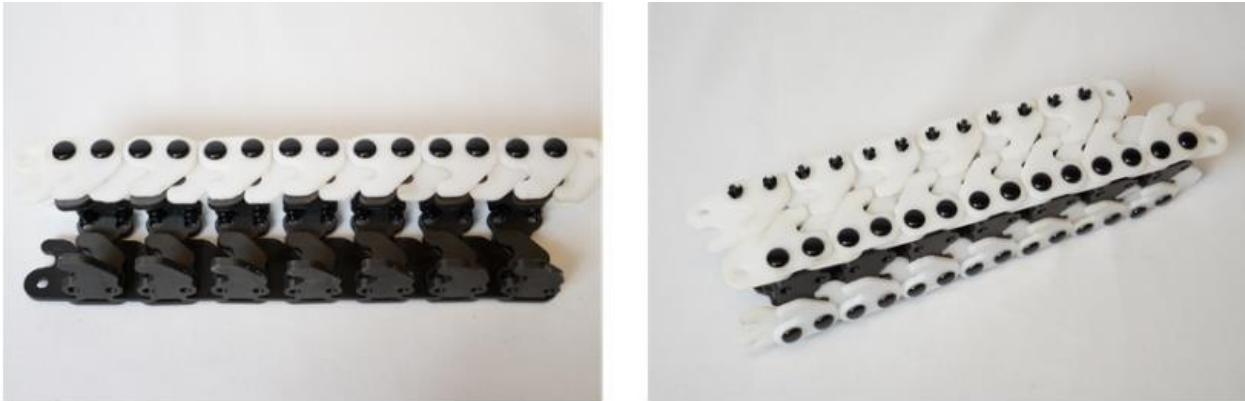
*Figure 37 (Top) Angled strand fabricated with snaps (Bottom) Angled strands fabricated for the rivet-based system shown zipping to a structure*

The flexibility of an angled strand, as well as a normal, linear strand, only allows motion in 1 degree of freedom. Though the strand can lock onto a pre-built edge, it can't close two edges together, as the pre-assembled edges would preclude a right-angled edge zipper from zipping into place. This prevents a closed channel from being assembled via angled pieces. Below several constructions are made using 3D printed pieces with snaps, including angled pieces.



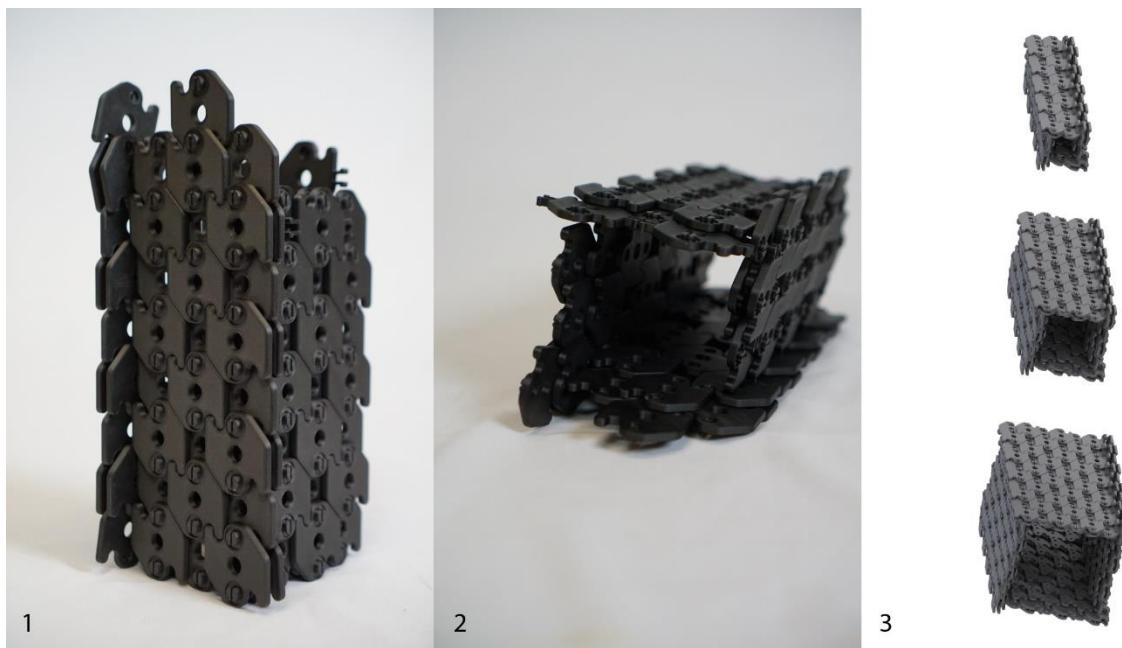
*Figure 38 Assemblages using angle pieces with the 3D printed snap system. Notice no closed forms can be assembled.*

As the teeth need to sandwich leading edges, when two leading edges come together to close an extrusion it is impossible to zip a strand in there without designing in enough play to the system for the parts to pull flex in multiple degrees along the spine. Below is an image showing a channel that only needs a right-angle edge to close it, and that same channel once the edge has been installed manually with flexible angled edge pieces. Flexible edges were developed to allow enough play for the two sides of an angled edge piece to pull apart and allow locking along both sides of a closed channel.



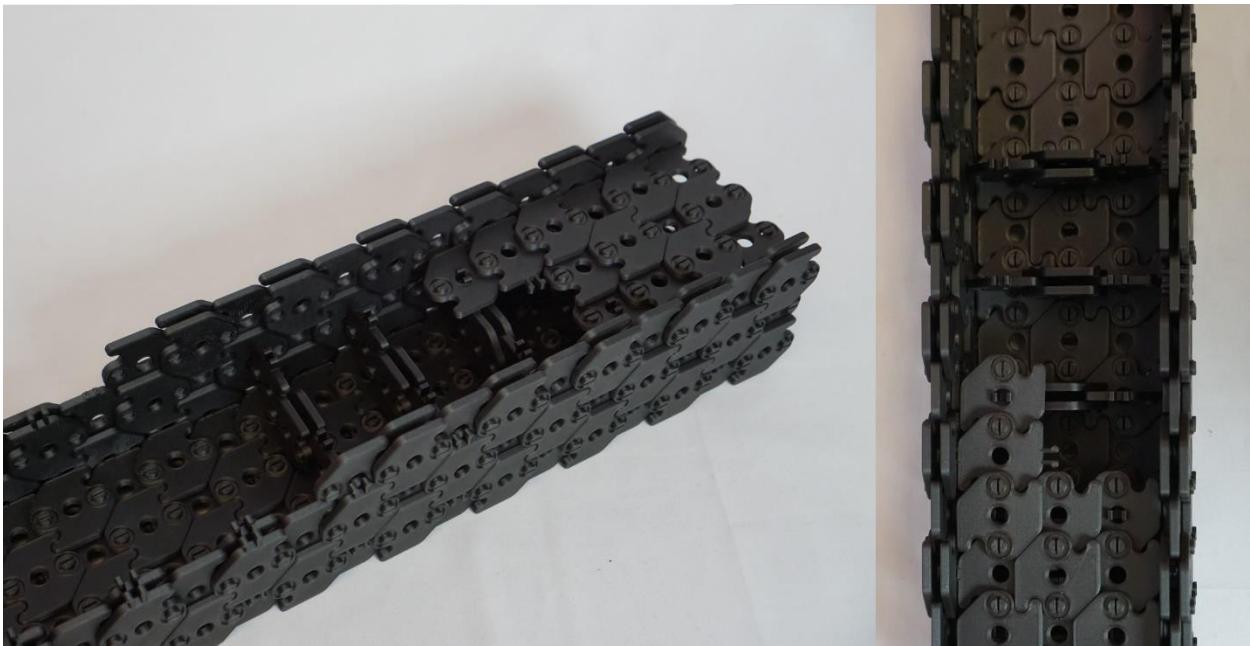
*Figure 39 Closed channel formed with angled edge pieces*

To form closed channels orthogonal pieces with side snaps must be used. The T joints created by connecting strands with side snaps allow boxed channels to be constructed as shown below. These channels can be varying sizes following  $width = strand\ width\ (1 + 2 \cdot n)$  where n is the number of strands. This distance needs to be an odd number of strands across as the side snaps from a middle tooth need to mate with a hole from an outer tooth.



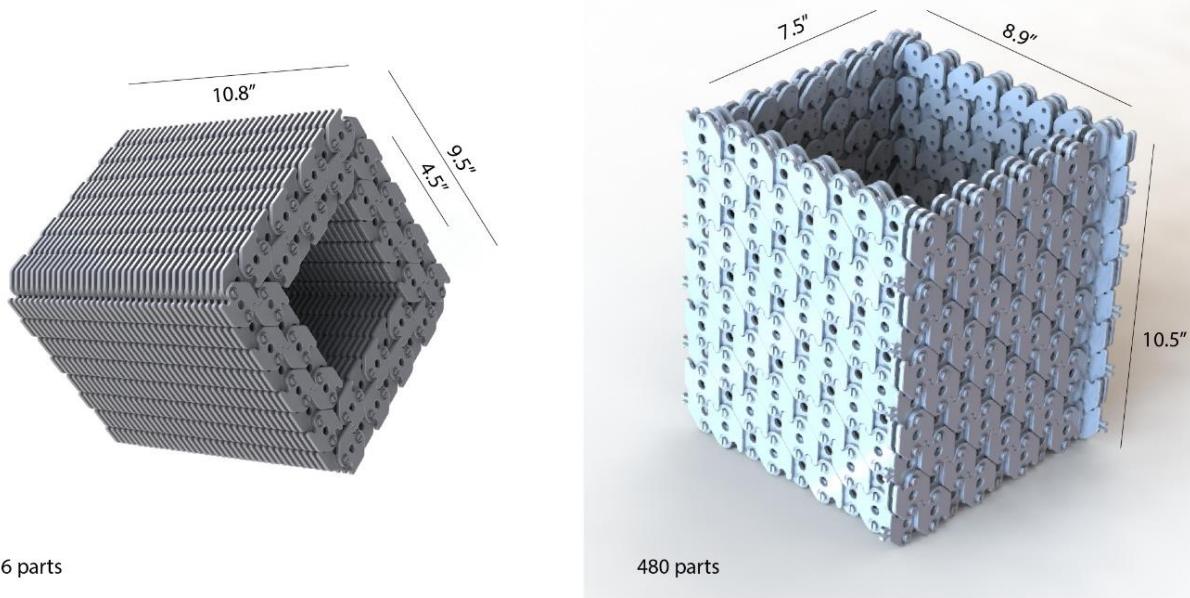
*Figure 40 Square channels made from orthogonal pieces with and without side snaps.*

Channels can have reinforced sections made from Zipped pieces that act as internal shear webbing as pictured below. Initial experimentation to demonstrate the structural improvements these cross braced pieces add are included in the Appendix.



*Figure 41 Channel with cross bracing*

Channels can be constructed by locking vertical plates using side snaps, or they can be fabricated by stacking rings of locked strands. Naturally, the difference in these design approaches yields different geometries. The renderings below show two of these different channel designs, labeled with length, external and internal dimensions as well as part count.



*Figure 42 Alternative channel designs (L) Stacked strands (R) Vertical sheets connected by side snaps.*

Combining flexible and rigid sections a closed bottom bucket was created. This is a capped-off, four wall channel, constructed using side snaps as shown above.



Figure 43 Bucket with flexible handles.

Combining channels as well as bent, branching and merging beams this chair was assembled to demonstrate construction of useful forms. Due to the limited number of pieces the seat of the chair is a wooden sheet, but it could easily be replaced with a Zipped sheet.



Figure 44 Picture of a chair made from Zipped parts

### 3.5 MOTION IN 3D OBJECTS

In addition to stationary structures the ability for these strands to be flexural in their un-locked configuration allows design and fabrication of flexible elements and mechanisms. Pictured below is a shark toy construction. As the tail and head of this shark are a single wide stack of pieces they remain flexible. The side fins are zipped onto the body of the shark; thus it is rigid.

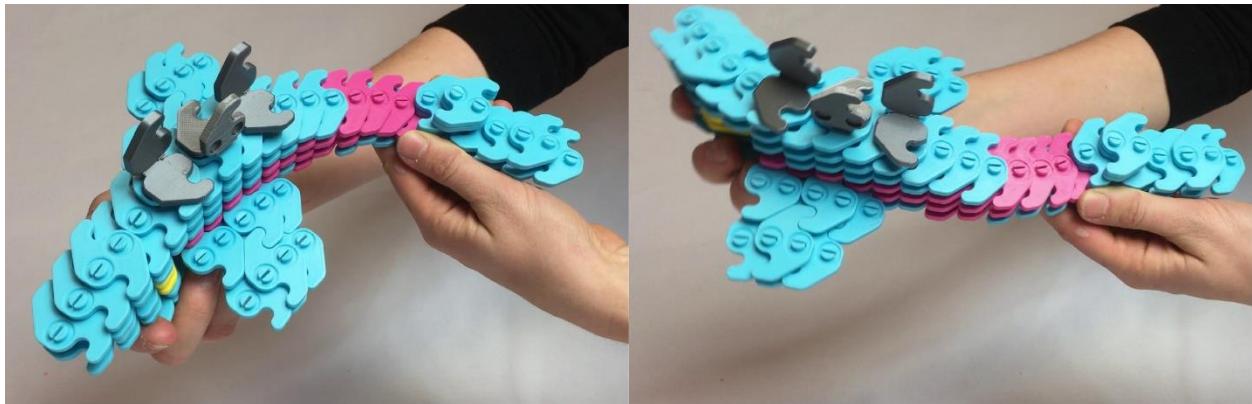


Figure 45 Zipped pieces used to construct a toy shark that bends in a fish-like manner

The figure below shows a single revolute joint constructed from Zipped pieces. A vertical bar is supported by a foundation constructed of Zipped pieces and contains the joint half way up its length. It can be tilted to a roughly 90-degree angle or supported vertically.

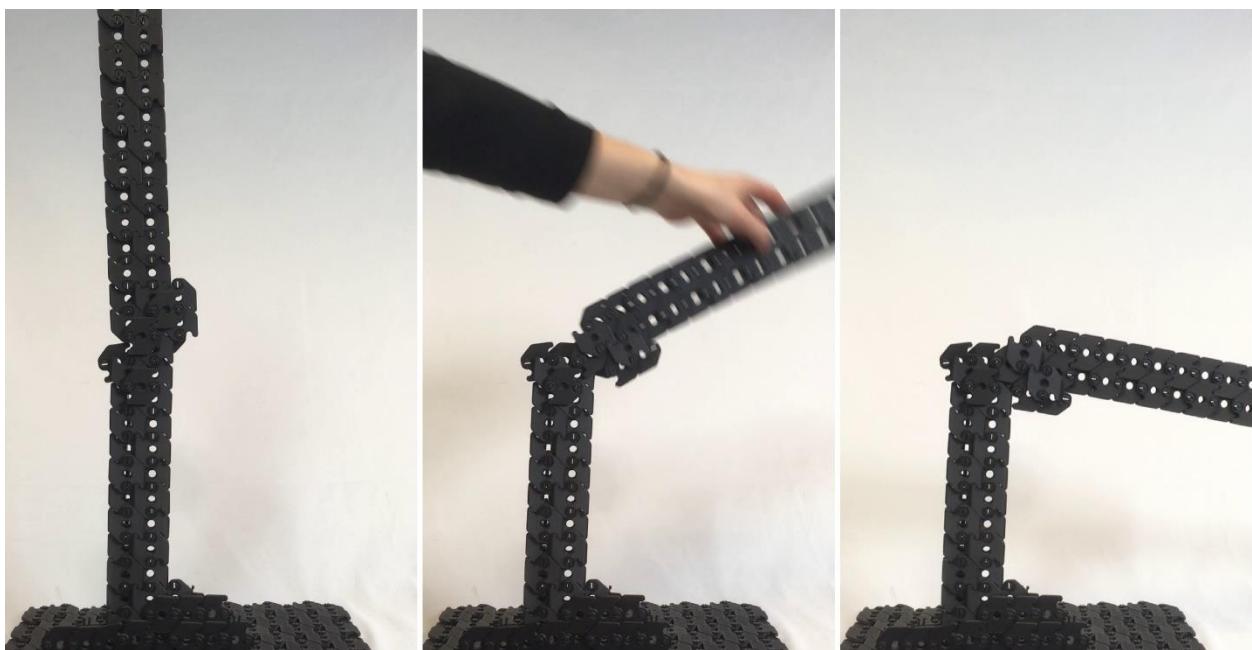


Figure 46 A single pivot point using stacked ZIPPED snaps as the basis for the revolute joint

The pieces are designed to allow 90-degree rotation, thus allowing strands to branch and merge. To create a wider angle of rotation multiple pivot points can be used in conjunction. For instance, the miniature crane shown below uses two pivot points.

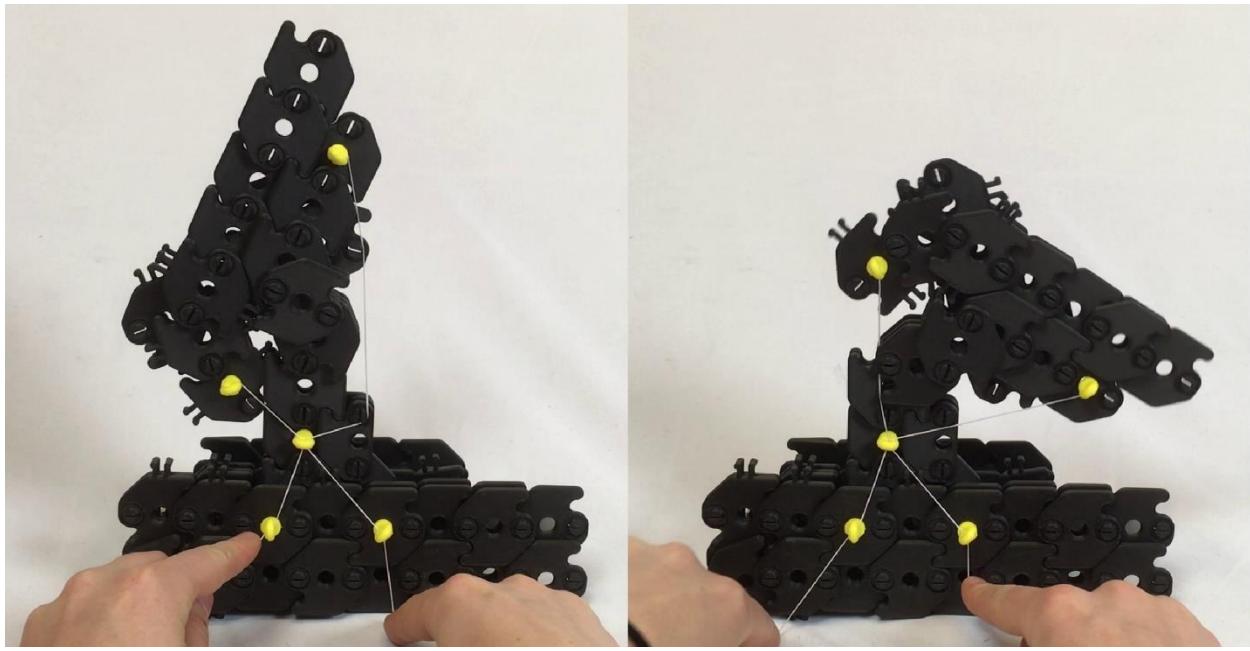


Figure 47 Mini crane designed with two revolute joints.

Additional mechanisms can be constructed using lengths of unlocked strand, thus having flexural sections. The figure below shows a miniature excavator that has manually operated tendons allowing controlled motion.

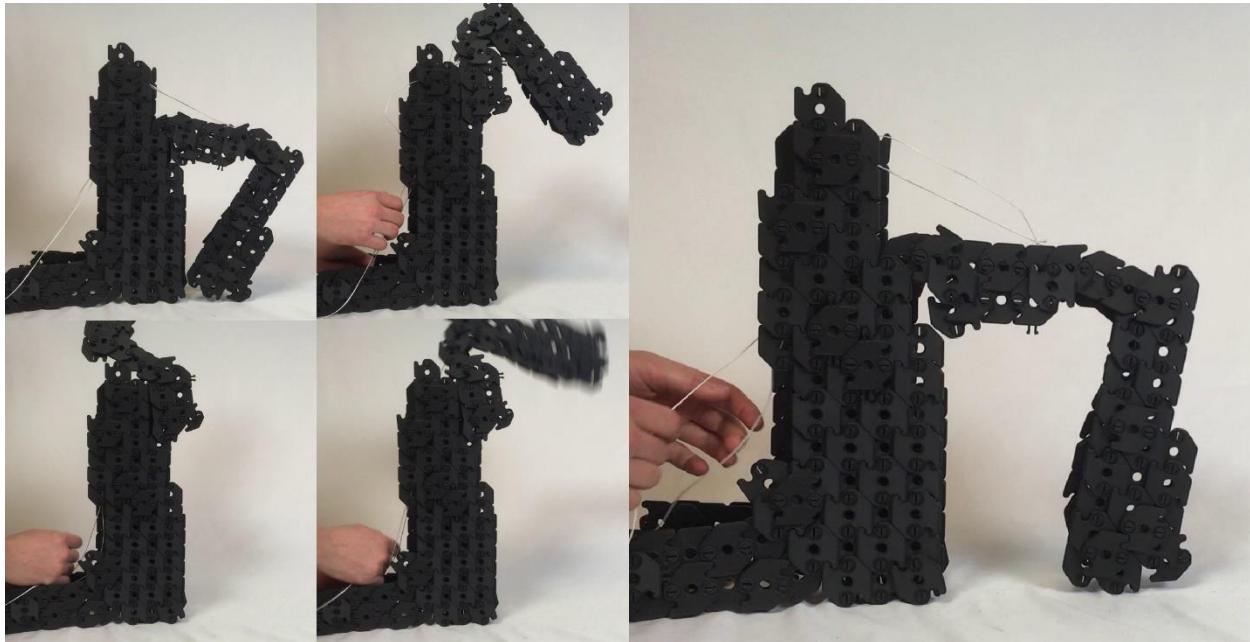


Figure 48 Mini-excavator showing flexural appendage with tendon driven actuation

## 4 MANUFACTURING ZIPPED

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Figure 49 2D laser cut sheets and 3D printed parts

The Zipped system aims to be an affordable method for robotic structural assembly. There are several methods of manufacturing these parts; including 2D and 3D approaches as shown above. The 2D and 3D manufacturing approaches differ in how the teeth are connected, though the fundamental geometry is identical. Consequently, strands made from either method will still zip together, though the teeth are not interchangeable between strands.

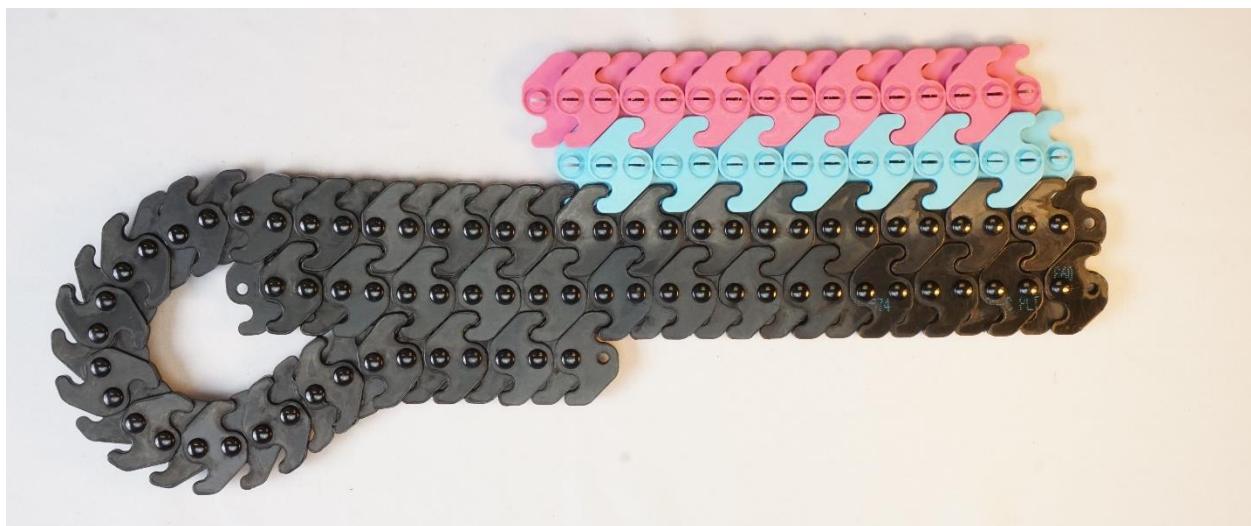


Figure 50 3D printed pieces connected to laser cut pieces showing their compatibility.

### 2D Manufacturing

2-dimensional parts without the inter-teeth connection are simple to manufacture via several methods, including laser cut, waterjet cutting and stamping, where the 2D part is cut from a flat sheet. Rivets or pins are used as the revolute joint. Thousands of prototype parts were laser cut and used with push-in rivets for experimental construction. Silhouettes of some of these parts are pictured below, showing their tessellating geometry in a flat sheet. The image shows the packing efficiency from a sheet of

fabricated parts. For the left two parts, there is only a small region of waste which comprises less than 10% of the sheet. This including the holes for the rivets, which is about 4% of the material waste.

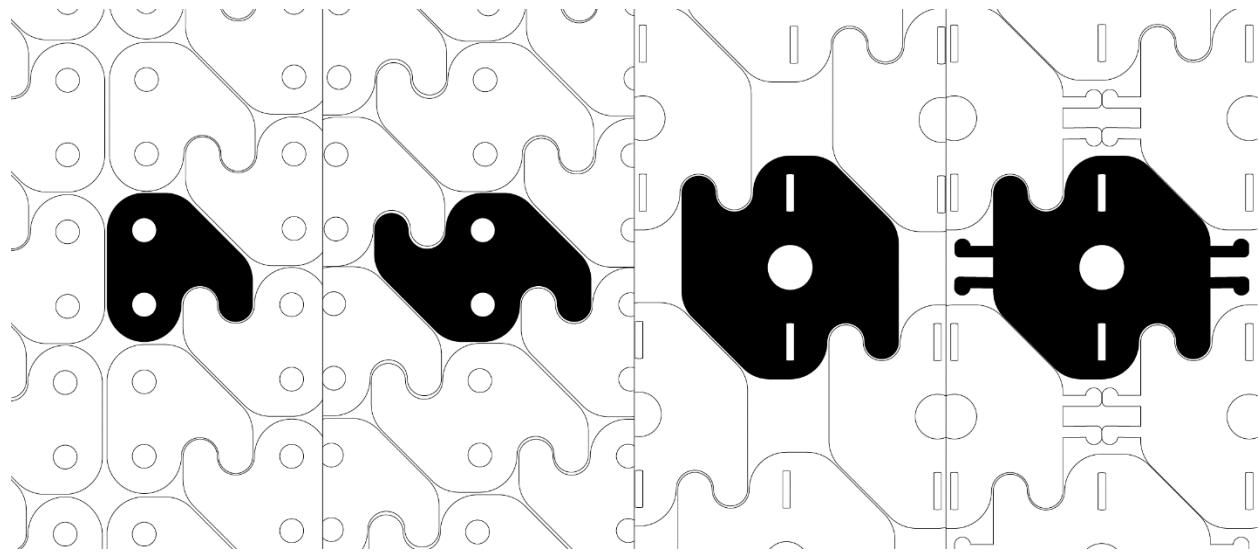


Figure 51 Part outline on a flat sheet showing tessellating geometry

Angled edges can also be cut from a 2D sheet and bent. Prototypes of this method have been made from steel but could also be made from plastic using heat for bending. Prototypes were also made with 2D profiles cut out from materials that include a living hinge, such as piano hinges.

A connector piece is required to connect 2D manufactured parts. Although rivets are a lower expense than the part material itself, they are still a non-negligible cost for this system as one is needed for every 3 parts. They cost between 0.12-0.24USD. This cost, and the inability to stack strands in the Z axis necessitated investigation into alternative methods to pin the teeth together. It is worthwhile to note however that zip ties are a fraction of a cent each. Though the zip-tie geometry is not ideal they preform the function required here, so with high enough volumes of production a zip-tie inspired connector part could be affordable.

### 3D Manufacturing

For 3D manufacturing the inter-teeth connection joints were altered from rivets to a snap fits. This design change allows an arbitrary number of parts to stack in the YZ plane. This part can be 3D printed without support material and creates a robust enough connection to support the rotational motion and torque on the parts as strands are zipped together. The consistency and structural integrity of these parts is not optimal when 3D printed however. The image below shows how the positive side of the snap can break along the layers of the 3D print. Failure rates seem highly variable among printed batches. In some batches every single part would fail, in others there were no immediate failures present. This is likely due to the filament used, but can also depend on the printer's settings, such as the z-offset height. To increase yield another iteration of design for manufacturing would be beneficial to increase feature size.

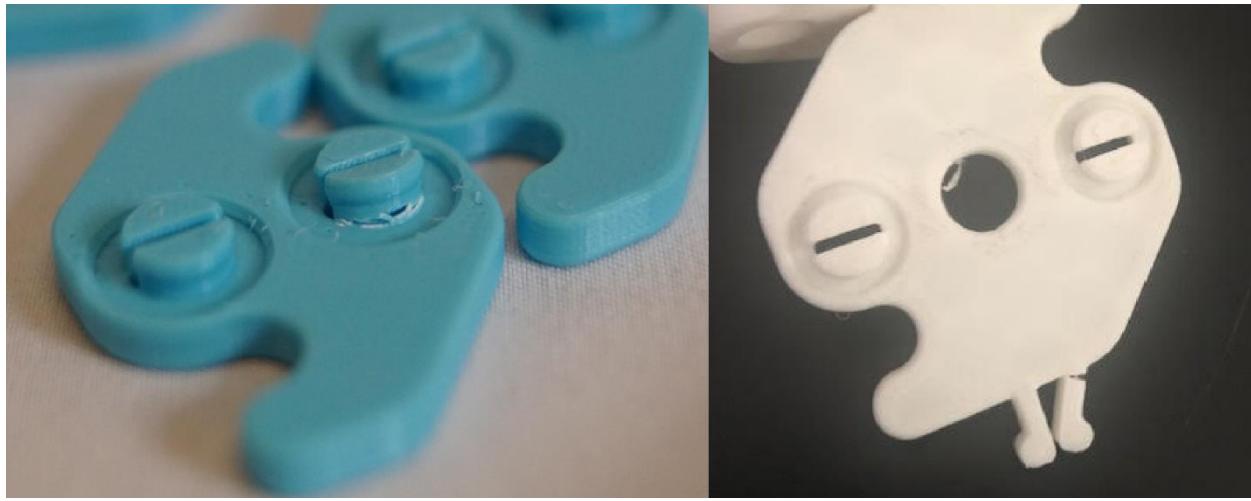


Figure 52 (L) 3D printed part failure along layer lines (R) Failure in side snap

3D printing allows rapid fabrication of parts with overhangs but does not scale for mass production as injection molding does. Ideally, these parts would be molded for large volume production. The images below, from Protolabs highlights undercuts and overhangs present in the snap fit design which need to be designed around for molding.

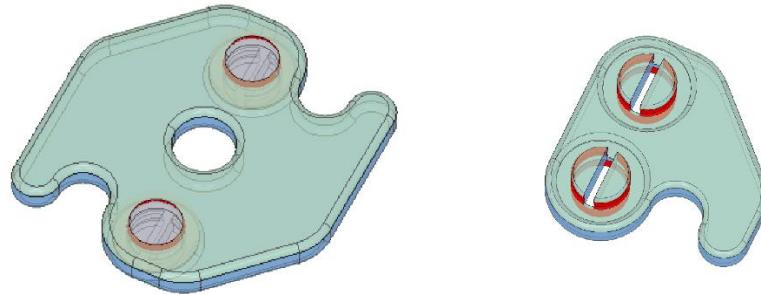


Figure 53 Molding analysis from Protolabs highlighting problem areas in red.

By decomposing the part and molding the pins independently from the body overhangs can be eliminated. The pins could be added using the custom jig shown below. It is essential the inter-part connection is stronger than the snap fit, thus an adhesive is applied to bond the materials. Quotes for molding these designs are listed in the Appendix.

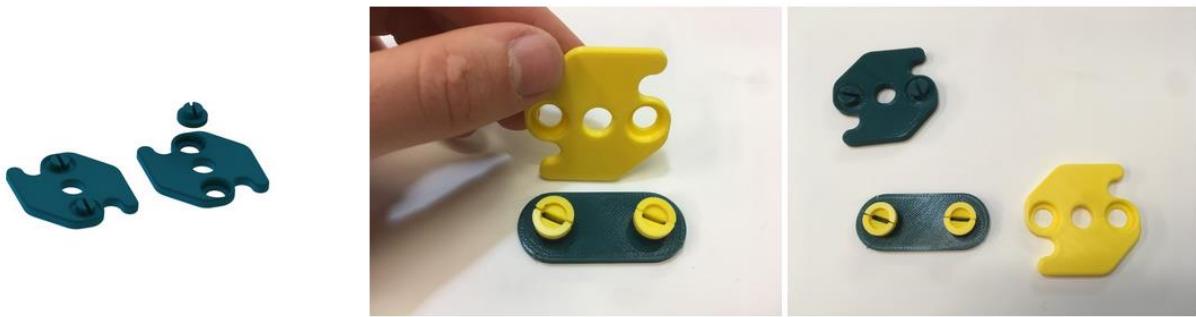


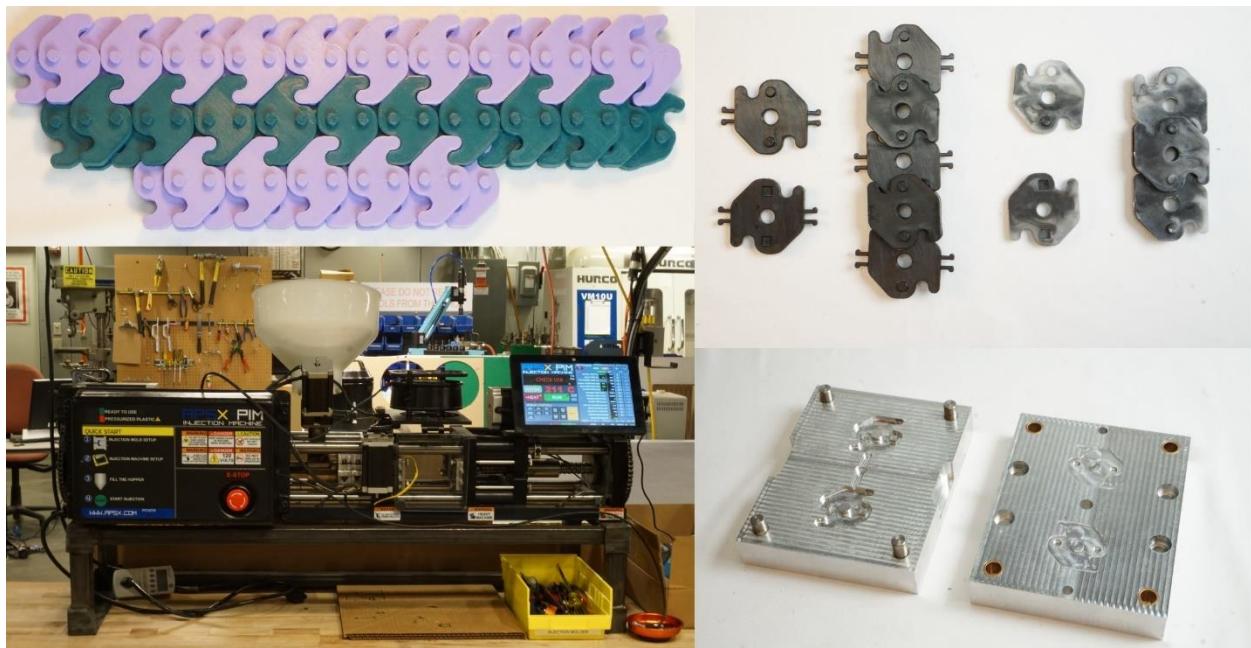
Figure 54 Multi-part molding and assembly.

There are several ways to manufacture these parts without the snap fit for the inter teeth connection, such as press fit or over molded metal snaps as discussed previously, but each has its draw back. Ultimately, for ease of manufacturing, and thus affordability, the parts below with press fit pegs were designed and molded.



*Figure 55 Parts after molding re-design. (L) Positive side with protruding pegs (R) Negative side showing cavity.*

Parts were molded in house on an APSX Plastic Injection molding machine using polypropylene pellets. Sample orthogonal parts with side snaps were also manufactured at Protolabs for comparison. Though the 3D prints of the models shown above functioned, allowing the parts to snap together and form strands that could zip and un-zip, the draft angle produced from the molded version prevented the peg press fits from holding. By re-machining the molds this fit could be adjusted.



*Figure 56 Molding experiments. Upper left: 3D printed parts designed for molding which contain no overhangs. Lower left: in-house injection molding machine. Lower right: aluminum molds Upper right: molded samples.*

The cheapest design with press fit pegs was priced with a unit cost of \$2.21 for 5000 units. In comparison Lego blocks are sold for about \$0.04 per piece. A 1kg spool of PLA costs ~\$40, and each part takes 1.5g, so 3D printing these parts is the most affordable option for low rate production.

## 5 MECHANICAL ANALYSIS

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Digital materials enable interchangeability of parts. By reducing fabricable elements to a set of fundamental building blocks elements can be added or alternated to tune the overall structural performance. Here, the mechanical locking of the parts is evaluated, the connection between parts that allows strands to form, as well as the structural performance of assembled strands treated as a bulk material. From experimentation an analytical description of the system can be summarized, enabling extrapolation and projection of structural performance with state-of-the-art materials as well as predictions for failure.

As this material system is anisotropic, it needs to be evaluated such that its response to a load in any direction can be understood, thus we will refer to the coordinate system shown below. Based on this coordinate system tension and compression tests were run along each axis to determine the materials moduli, allowing its off-axis moduli to be described as via a mathematical transformation. The axes depicted define planes which are used to describe the shear forces.

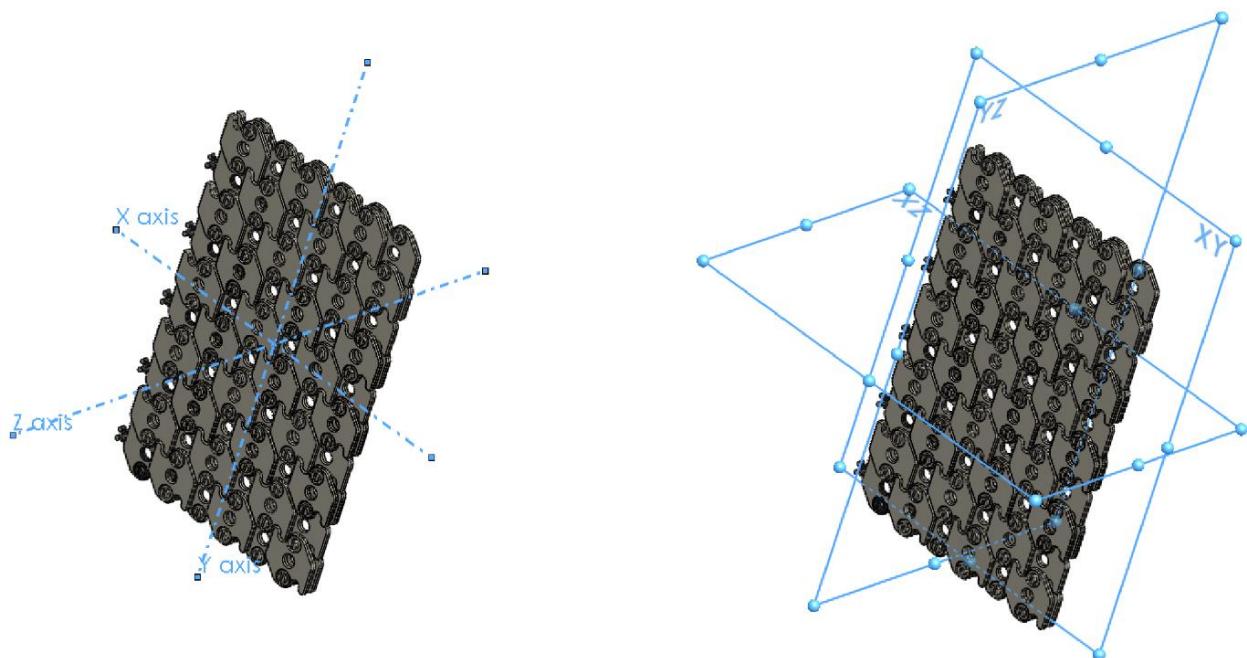
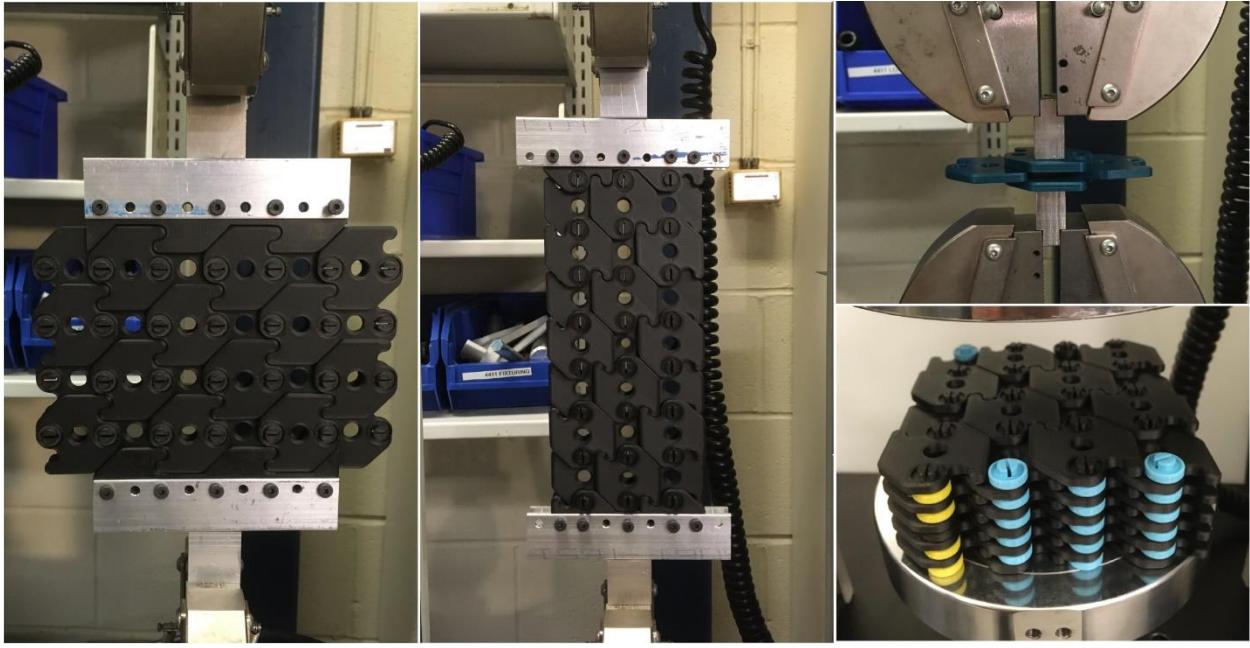


Figure 57 Sample with labeled axis and coordinate system planes

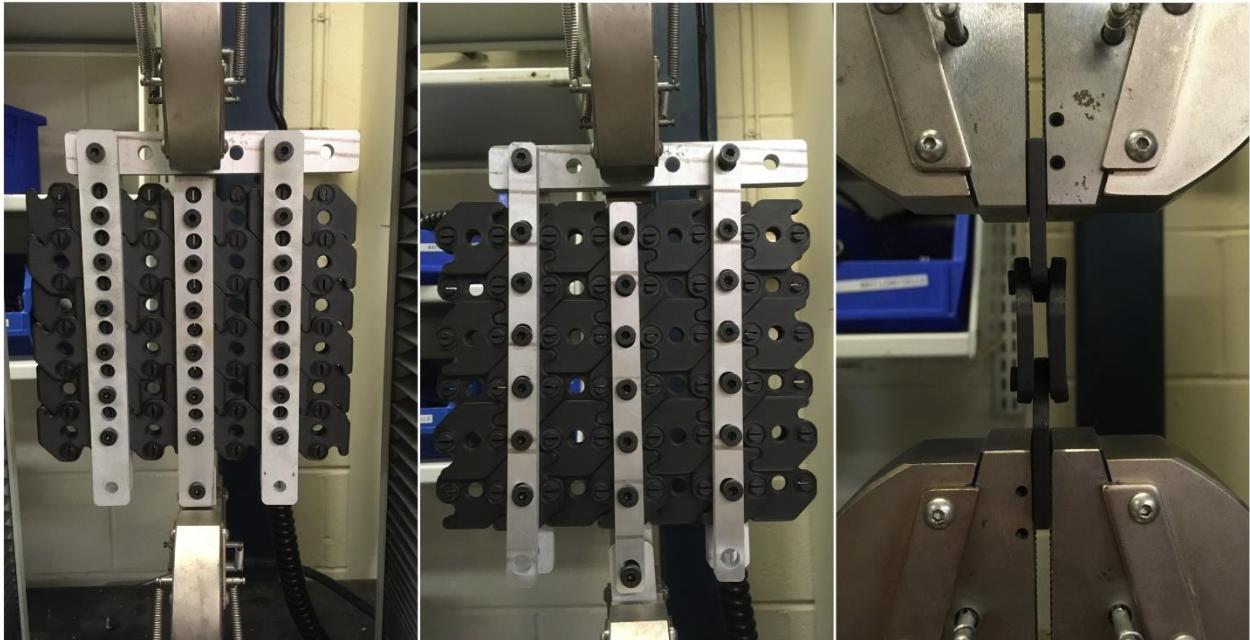
Multiple experiments were run on different with various Zipped material systems and it was found that MarkForged 3D printed Onyx parts performed with the most repeatability.

Experiments were done to characterize the material as assembled into a uniform sheet from MarkForged 3D printed Onyx parts. The experimental set up showing custom holding jigs for each axis are pictured below.



*Figure 58 Experimental set up for material characterization along each axis. From left to right: X, Y, (top) Z tension, (bottom) Z compression*

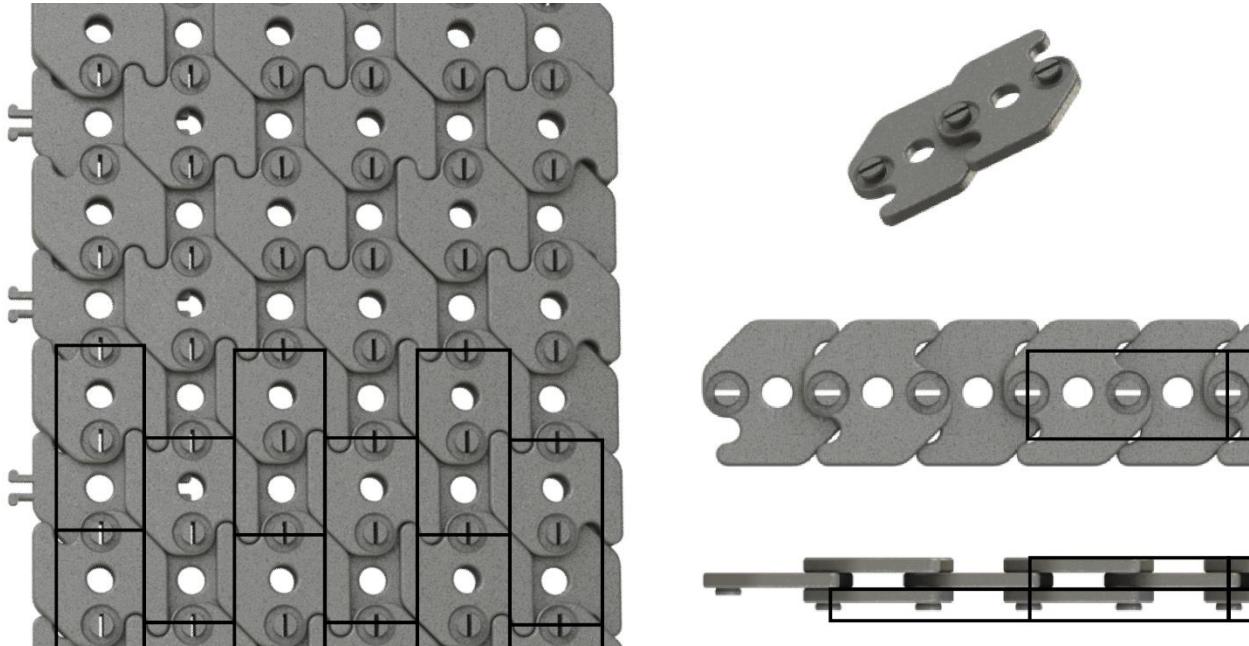
Shear was also evaluated alone each planar axis. The experimental set up for shear measurement is shown below. Aluminum bars were fabricated to fit around the snaps and pinned along the central holes in the parts. These slats make light contact with the sample, but no pre-load was applied.



*Figure 59 Experimental set up for material characterization on shear planes. From left to right: YZ, XZ, XY*

As force and displacement were measured by the Instron, stress and strain were calculated by dividing the values by sample cross sectional area and length. The smallest tileable unit, two offset pieces, is

shown in the image below. The bounding box for this unit is 1"x2"x0.3". The face of this bounding box is tiles to the sample size and used as the cross-sectional area for scaling these samples.



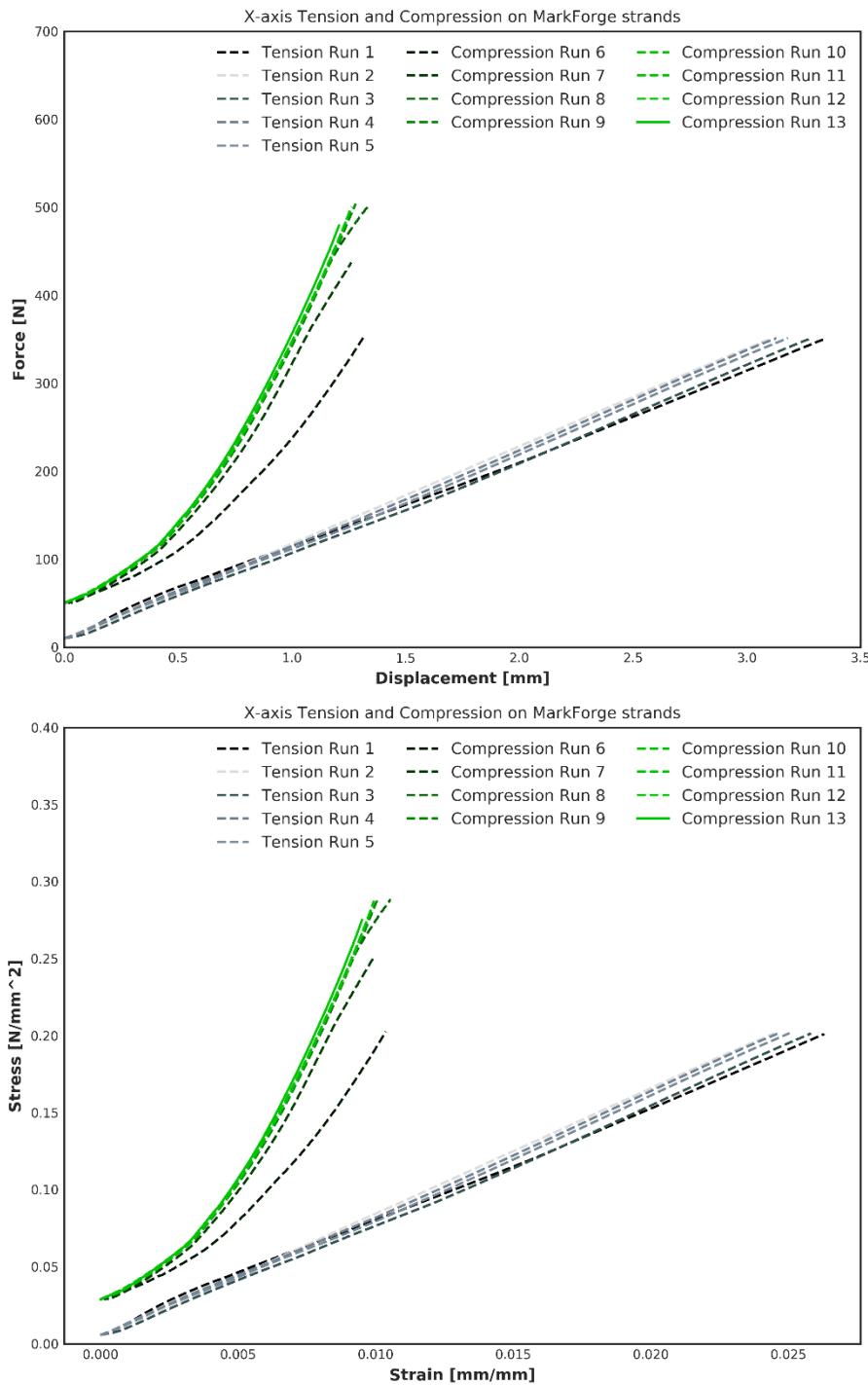
*Figure 60 The smallest tileable unit of the Zipped tessellating geometry is the two pieces shown on the upper right. The bounding box defined by this unit is superimposed above on an assembled sheet on the left, as well as on a single strand shown on the bottom right.*

The first chart for each of the following sections shows the force and displacement measured on the sample. Linear looking sections of the force and displacement data were selected for linearization. They were cropped based on a pre-load force value, then a limit on the delta in displacement from that pre-load. These limits used are listed in the tables following the experimental results.

*Table 1 Properties of raw materials used for reference*

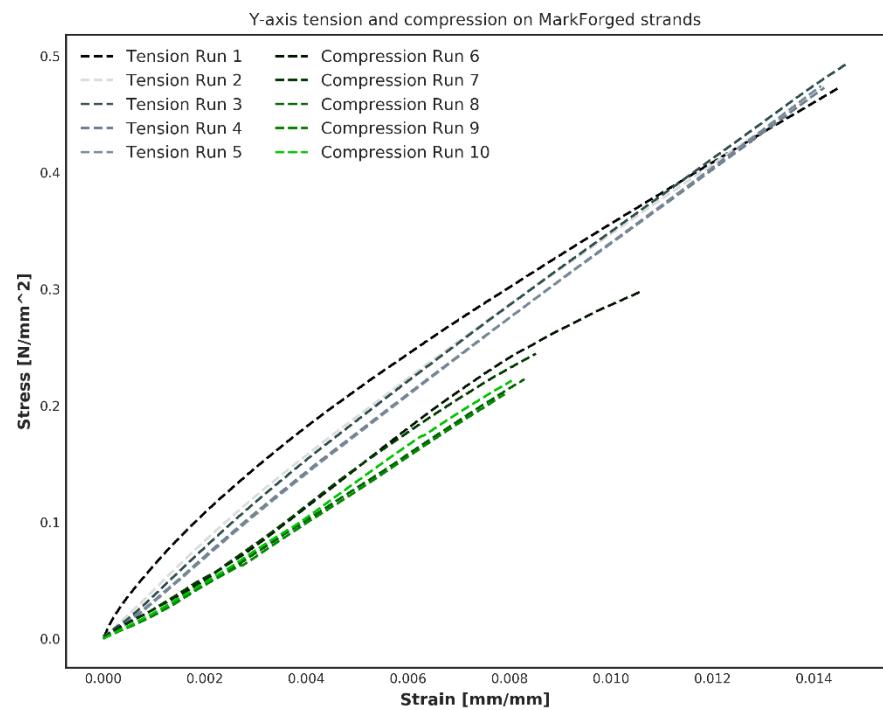
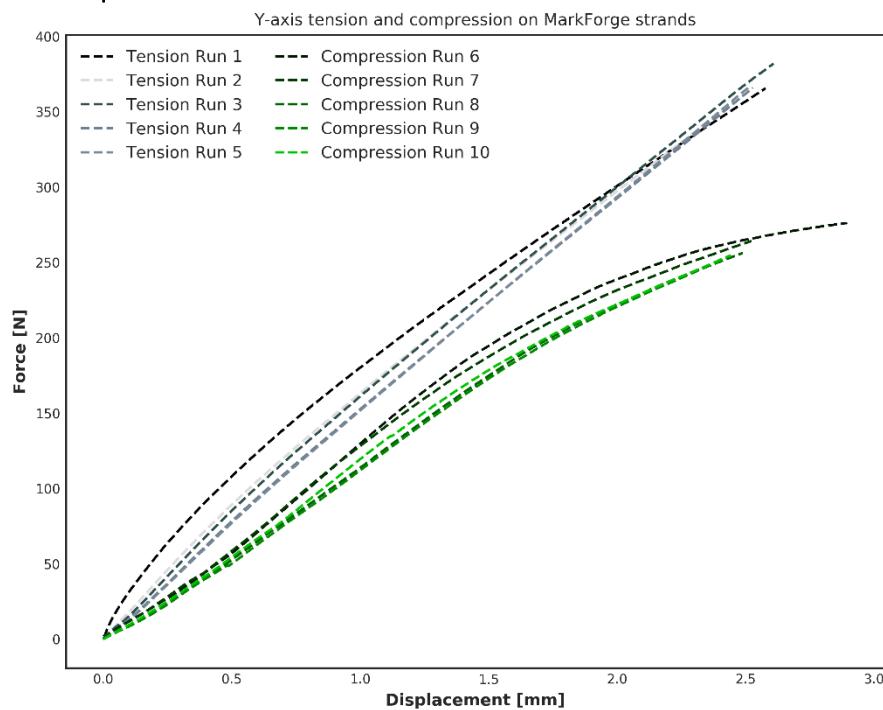
Material	Tensile modulus [GPa]	Tensile stress at yield [MPa]	Shear Modulus [GPa]
ONYX [30]	1.4	36	
PLA [27]	2.3	49.5	
Delrin [31]	3.2	71	
6000 Aluminum [32]	69	276	26

## X axis tension and compression



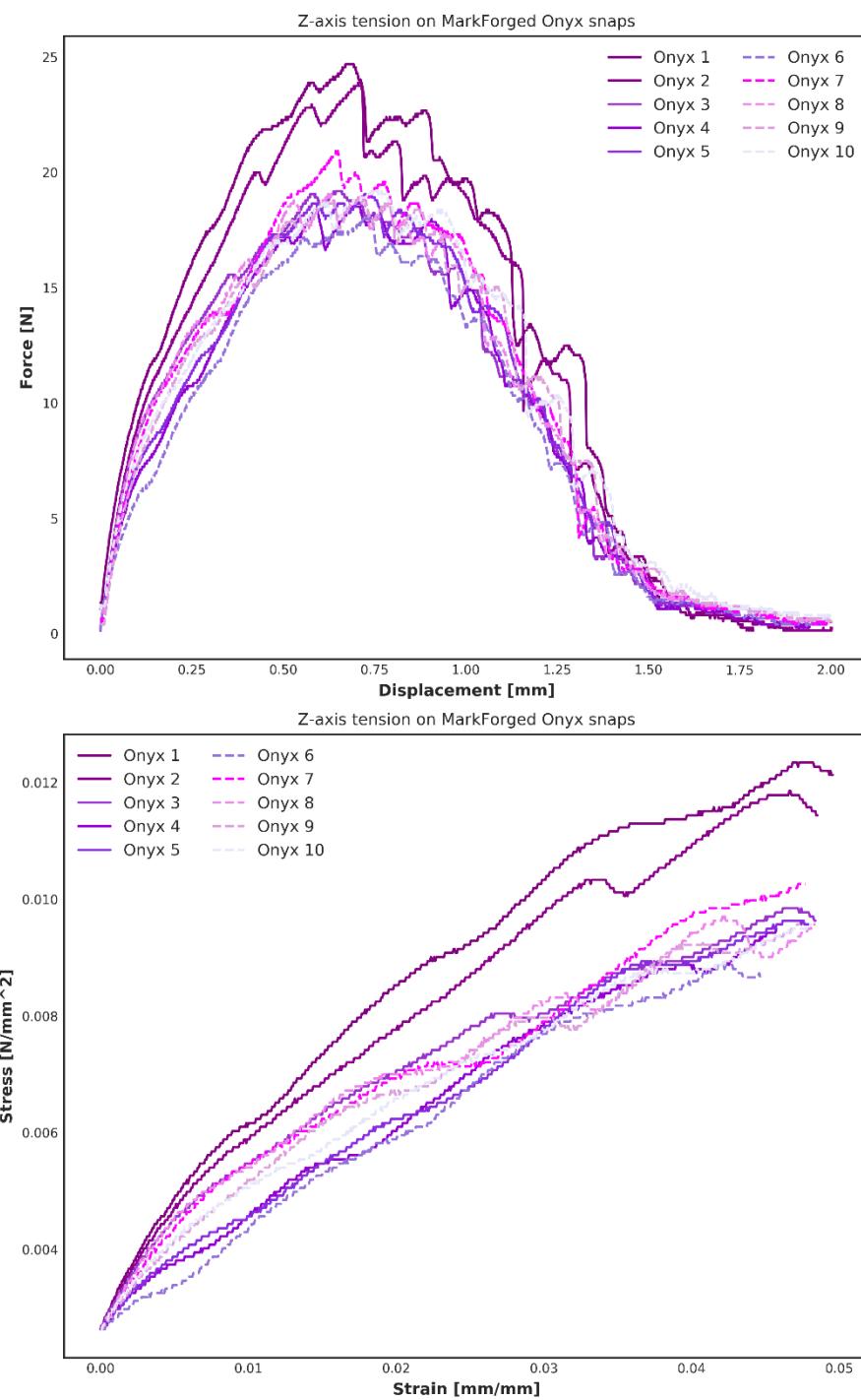
Limit	Tensile value	Compressive value
Sample area	871 mm <sup>2</sup>	871 mm <sup>2</sup>
Sample length	127 mm	127 mm
Pre-load	10 N	-50 N
Displacement limit	none	-3.25 mm
Measured modulus	$7.8 \pm 0.3$ MPa	$24.4 \pm 3.2$ MPa

## Y axis tension and compression



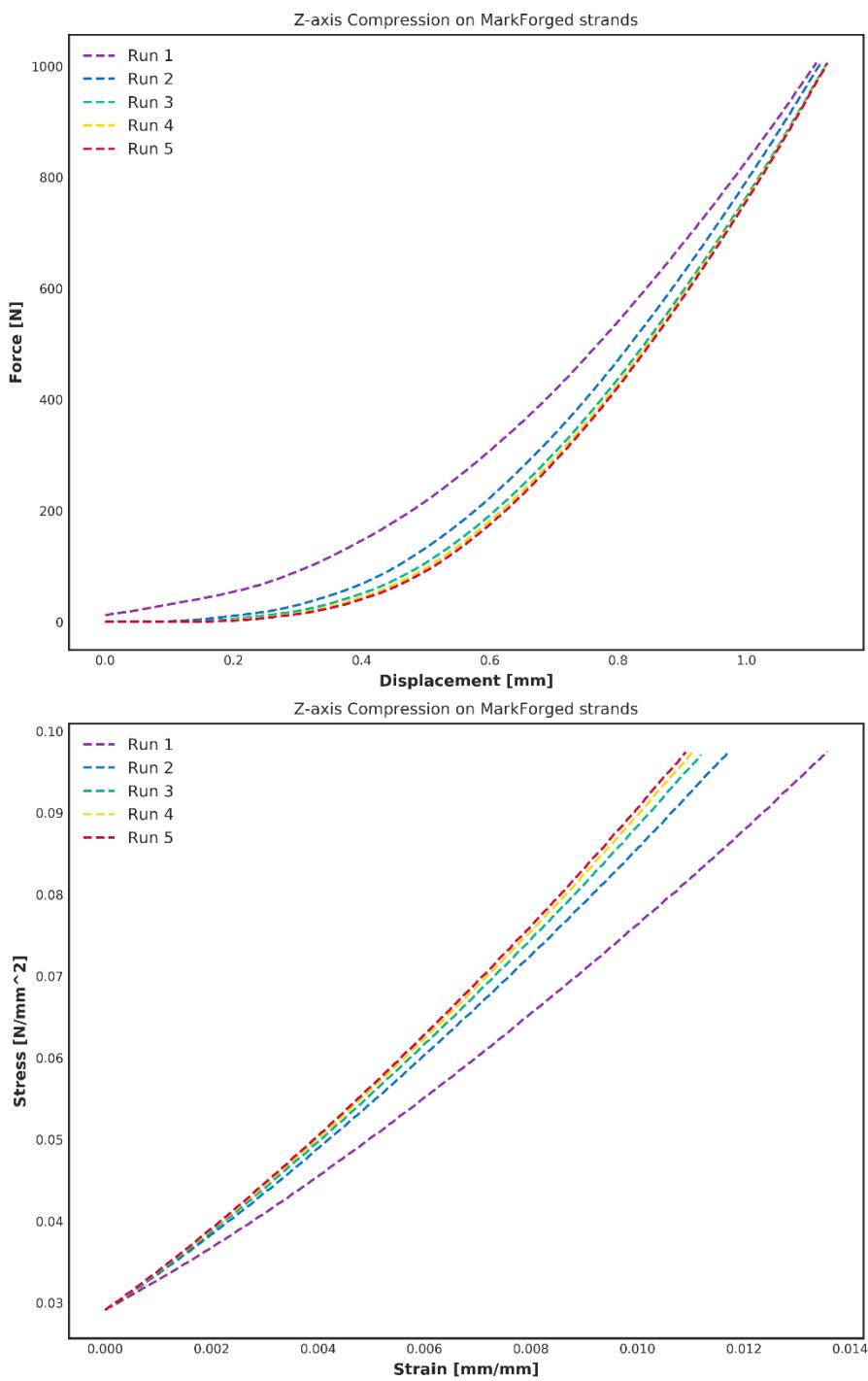
Limit	Tensile value	Compressive value
Sample area	774 mm <sup>2</sup>	774 mm <sup>2</sup>
Sample length	178 mm	178 mm
Pre-load	0 N	0 N
Displacement limit	none	2 mm
Measured modulus	$32.6 \pm 1.3$ MPa	$28.6 \pm 1.3$ MPa

## Z axis tension



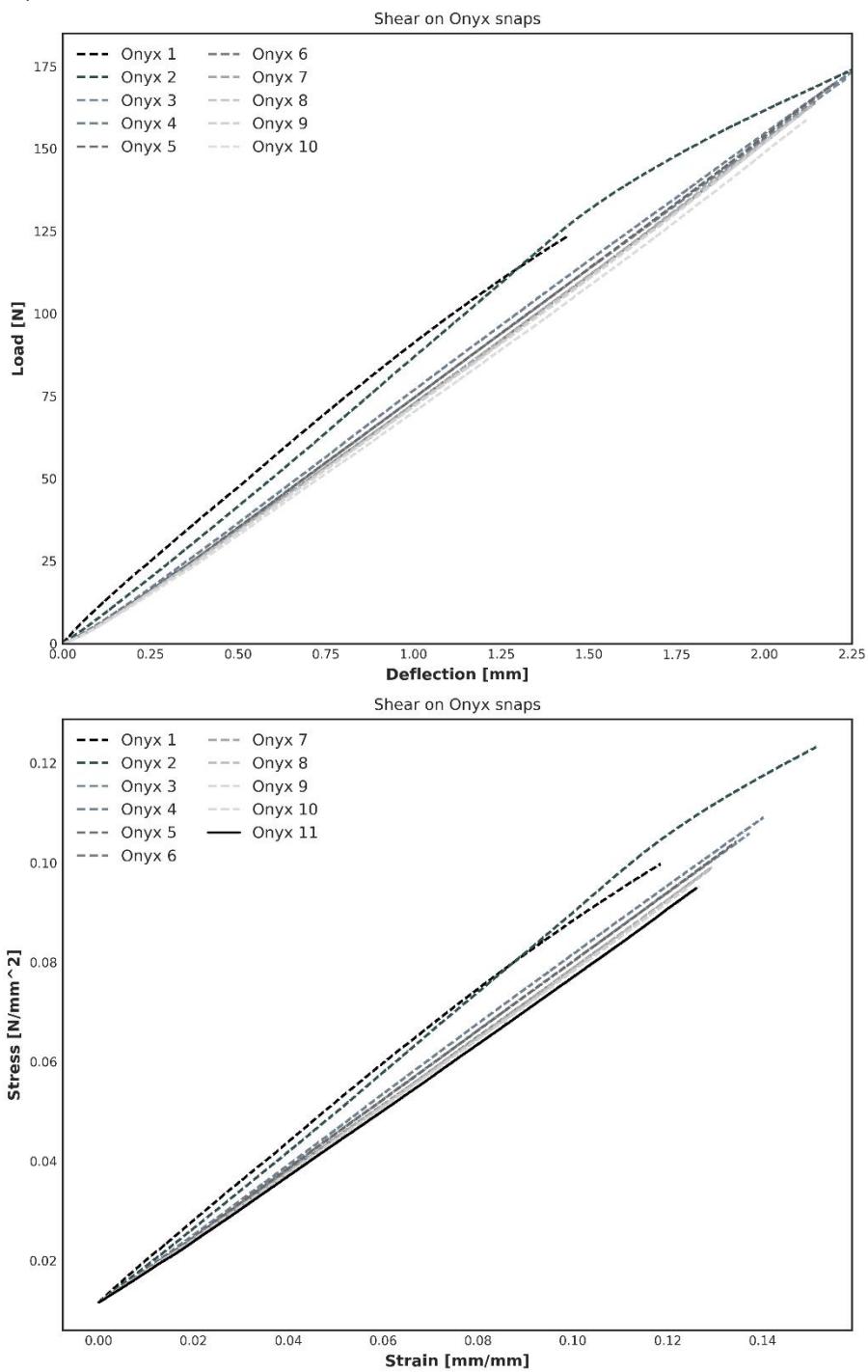
Limit	Tensile value
Sample area	1935 mm <sup>2</sup>
Sample length	11.4 mm
Pre-load	5 N
Displacement limit	0.6 mm
Measured modulus	0.15 ± 0.02 MPa

## Z axis compression



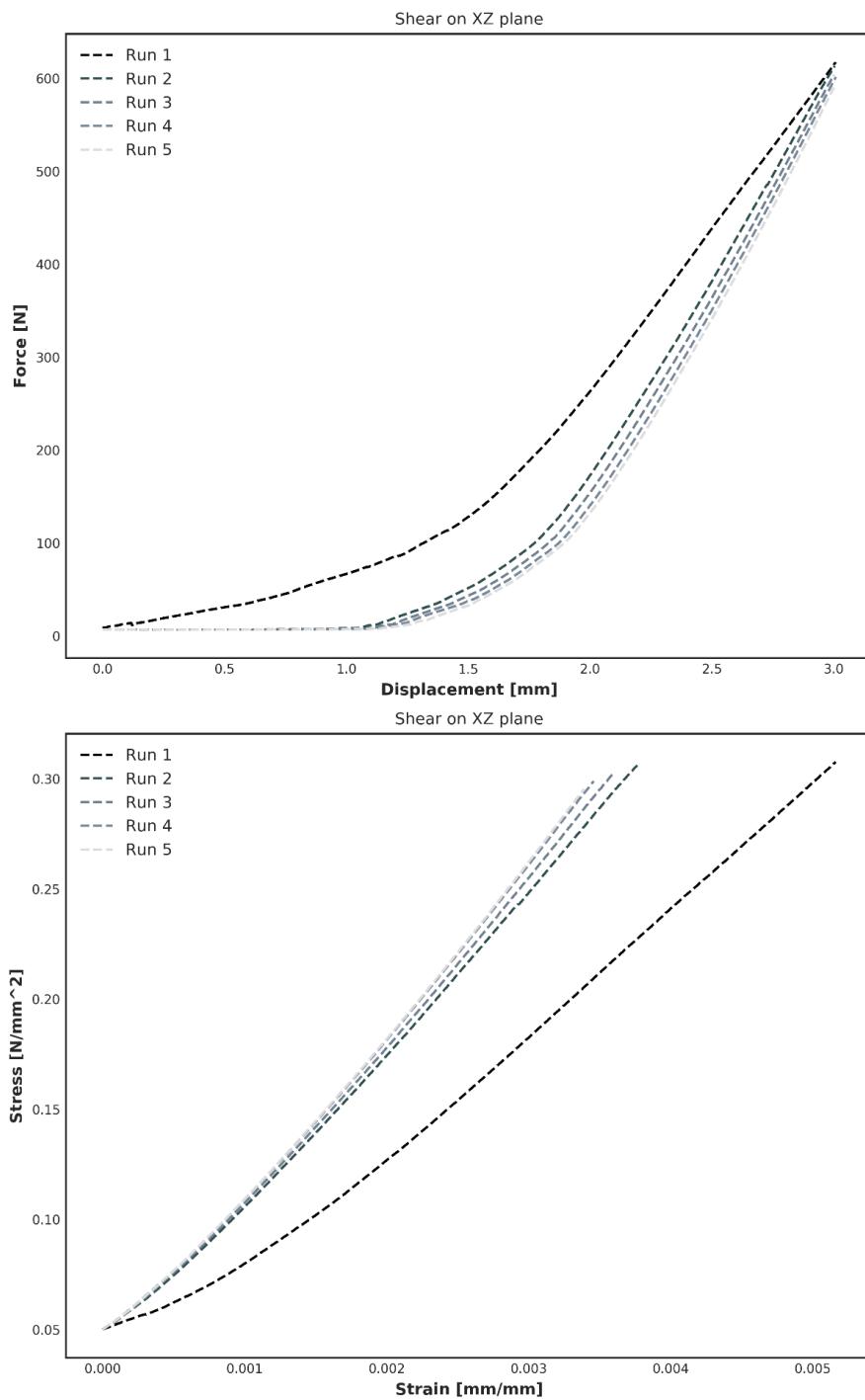
Limit	Compressive value
Sample area	10323 mm <sup>2</sup>
Sample length	38.1 mm
Pre-load	300 N
Displacement limit	none
Measured modulus	5.9 ± 0.5 MPa

## Shear on the XY plane



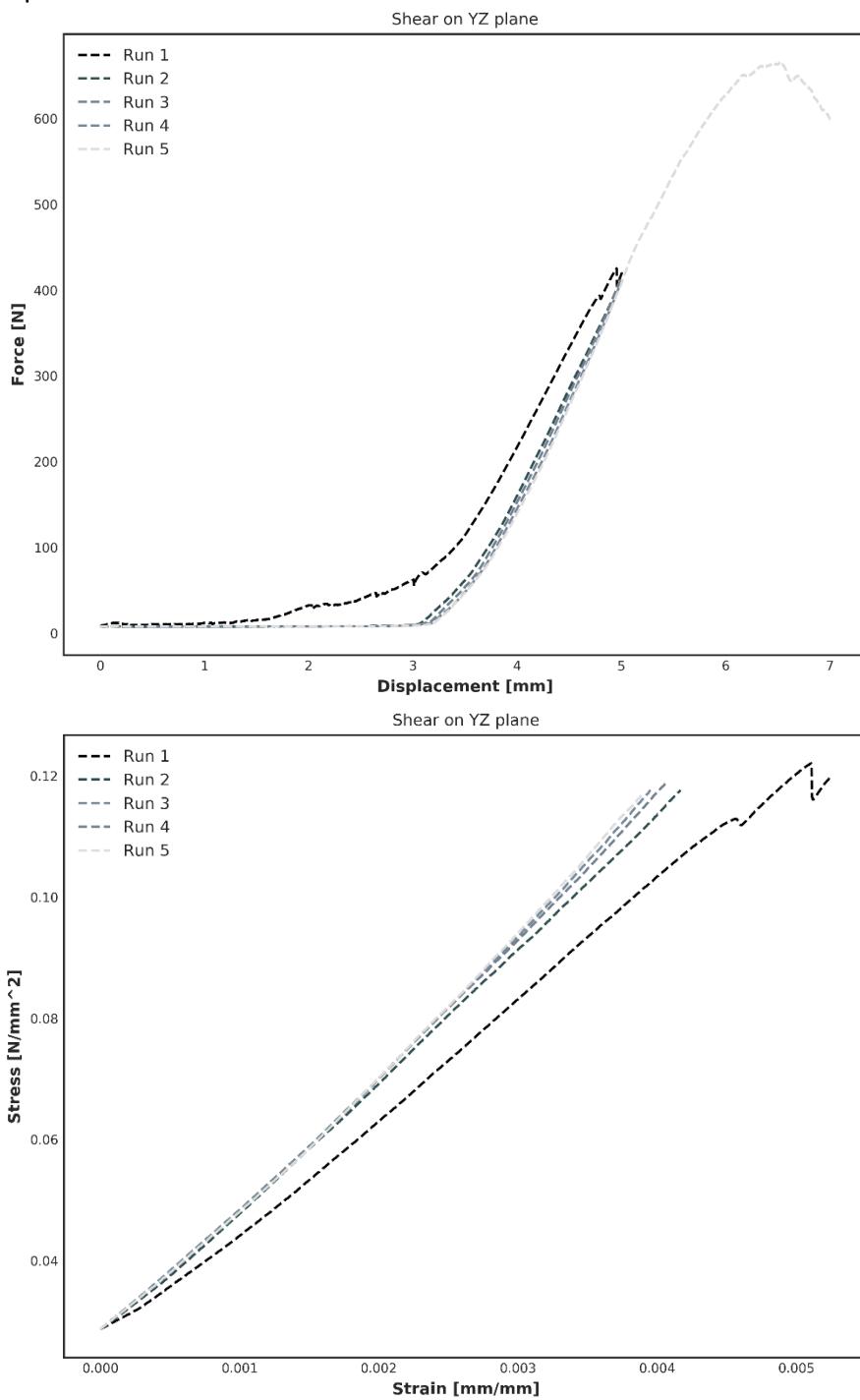
Limit	Value
Sample area	1290 mm <sup>2</sup>
Sample length	11.4 mm
Pre-load	15 N
Displacement limit	2 mm
Measured modulus	0.6 ± 0.2 MPa

### Shear on the XZ plane



Limit	Value
Sample area	1742 mm <sup>2</sup>
Sample length	152 mm
Pre-load	100 N
Displacement limit	3 mm
Measured modulus	36.4 ± 4.8 MPa

### Shear on the YZ plane



Limit	Value
Sample area	1741 mm <sup>2</sup>
Sample length	152 mm
Pre-load	100 N
Displacement limit	5 mm
Measured modulus	21.6 ± 1.6 MPa

The results from each axis are summarized below. For reference the axis and shear planes are labeled in the following image, as well as reference material properties in Table 1.

*Table 2 Material Properties by direction for MarkForged 3D printed Onyx snap pieces*

Direction	Tensile modulus	Compressive modulus
X	8 MPa	24 MPa
Y	33 MPa	29 MPa
Z	0.15 MPa	5.9 MPa

Shear plane	Shear moduli
XZ	36 MPa
YZ	22 MPa
XY	0.6 MPa

These results show that the material behaves in an anisotropic manner. The resulting deflection from applied loads varies depending on what orientation the load is applied from. For instance, shear in the XY plate was shown to be 60 times less than shear in the XZ plane. Some of this variability may be due to experimental set up, as the XY plane was loading only a single snap, whereas the experimental set up for XZ and YZ shear measurements incorporated several strands. Though that is the most extreme difference, variability exists between all the orientations tested. This anisotropy means structures build from the Zipped system need to consider the materials orientation and incorporate that into the design to optimal structural performance.

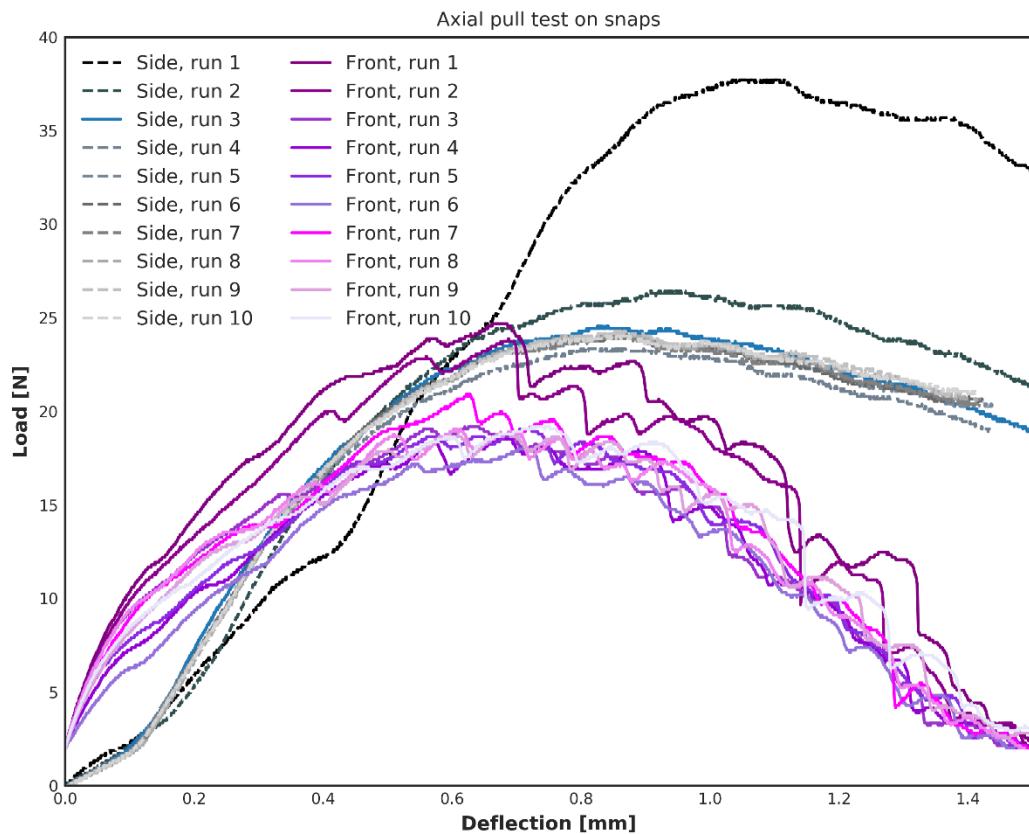
## 5.1 SNAP EVALUATION

The connection between parts is a critical part of the system design. This connection needs to be made to secure the parts to each other, but also needs to remain reversible. The strength of this connection determines the overall structural performance, therefore the connection between the teeth and the parts is evaluated and quantified independently and individually by connection type. Results show maximum average and standard deviation for axial, tensile tests on the front versus side snaps printed with MarkForged Onyx. The following graph compares side and front snaps on the same scale.

Onyx side snap averaged maximum load  $27.1 \pm 5.5\text{N}$

Onyx front snap averaged maximum load  $20.2 \pm 2.2\text{N}$

These results demonstrate the side snaps are similar strength to the face snaps, though experimental results also show more critical failures in the side snaps, particularly when printed in PLA.



## 5.2 BEAM BENDING

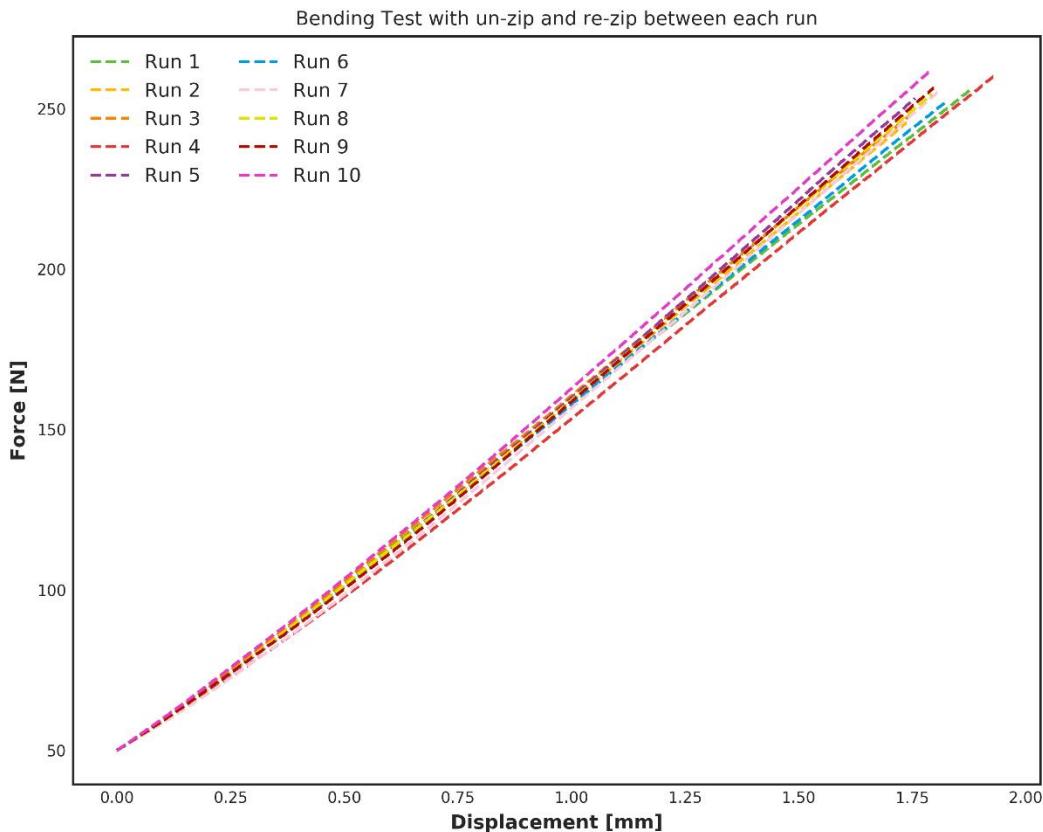
The Euler-Bernoulli equation, shown below, describes the deflection of a beam,  $\delta$ , under load,  $P$ . For these 3-point bending experiments the formula can be adjusted to the following, where  $I_x = \iint x^2 dxdy$  and  $\frac{P}{\delta_P}$  is measured on the Instron.

$$E = \left(\frac{P}{\delta_P}\right) \frac{L^3}{48I}$$

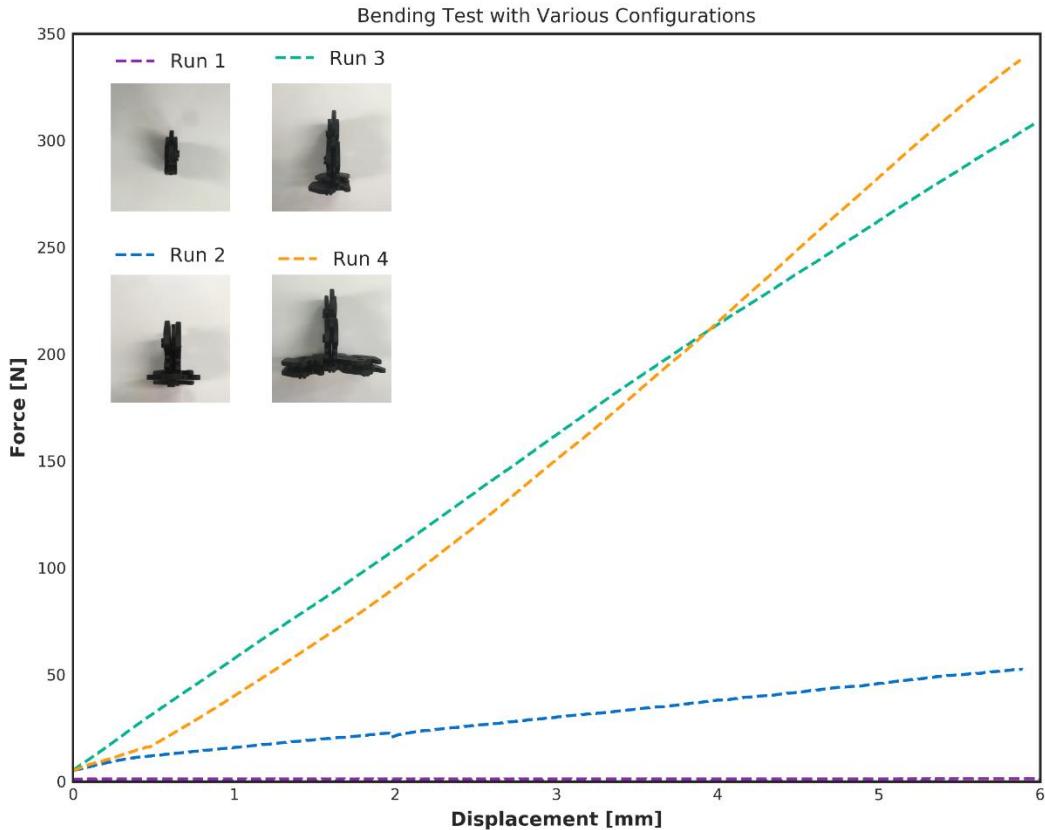
Results show the ability for the system to zip, un-zip and re-zip without altering its structural performance. Below are a 10 teeth thick stack, 12" long was zipped, loaded and un-zipped and re-tested 10 times. Using a 50N pre-load the results for force over displacement show only a 2.5% standard deviation between samples run. This small standard deviation demonstrates the materials are not altered by the zipping process.

$$114.9 \pm 2.8 \text{ [N/mm]}$$

The second polar moment for this sample is  $1.56 \text{ in}^2$  therefor the elastic modulus measured here is found to be 13.0 MPa, which is within an order of magnitude of the Y-axis tensile modulus, 33 MPa, measured above. This mismatch in results could be due to the samples thickness to length ratio being too large for an Euler-Bernoulli model to be accurate. Another method which accounts for shear deformation, such as Timoshenko's beam model, may be yield closer results [33].



Several experiments were performed on different geometry beams demonstrating the ability to architect structural difference, as with any beam design, but here it is via an assembly process. Below is a representative example showing the ability to increase the stiffness of a beam by changing its geometry. Notice Run 1, a single strand supports effectively no load. It is interesting to note Run 3 and Run 4 have similar stiffnesses though Run 4 has about 40% more material than Run 3. Pictured are the profiles of the beams tested in bending.



There are many areas where the parts in this system interact causing the beam to stay rigid which do not affect the internal material properties of the parts. The face of each zippered tooth wears on its neighbors, exacerbated by any pre-load caused by the rivets. Space between the hole for the pin and the rivet causes slop, where the beam can freely bend back and forth before engaging with the pin. Shear as the parts mate and slide on each other also causes play. For our models all these effects are grouped together and accounted for by as part of the material system, thus, any variation in the assembly of the teeth alters the material system, for example switching aluminum to polypropylene rivets will alter the model, even if the teeth are identical. Similarly, adding more pre-load to the rivets will affect the model, necessitating new experimentation, even if the material remains consistent.

## 6 ZIPPED AUTOMATION

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The Zipped system is designed for hierarchical assembly; 0-dimensional pieces assemble into 1-dimensional strands which zip into 2 and 3-dimensional forms. Construction requires automation of these steps, as well as a mechanism by which the parts arrive to the build site. Zipped can be assembled into rigid and flexible segments, providing an advantage over traditional assembly blocks, which often only hold form under a pre-load. Assemblages of flexible and rigid segments allowed mechanisms and robots to be made using the Zipped for their structure. Additional robots were designs and fabricated traditionally that travel along the structure and interact with it in an enzymatic way.

Below two prototypes are pictured. On the left a robotic ring fabricated from Zipped is shown which can bend and crawl on a Zipped structure. Pictured below on the right is a robotic carriage fabricated using traditional processes which can roll up two strands zipping them together. The fundamental difference between a robot constructed using Zipped parts and one constructed from foreign media is the future ability for the system to assemble recursively. If Zipped can create robots from Zipped parts which can create other robots, then these robots can assemble themselves and thus fabrication speed can exponentially increase.

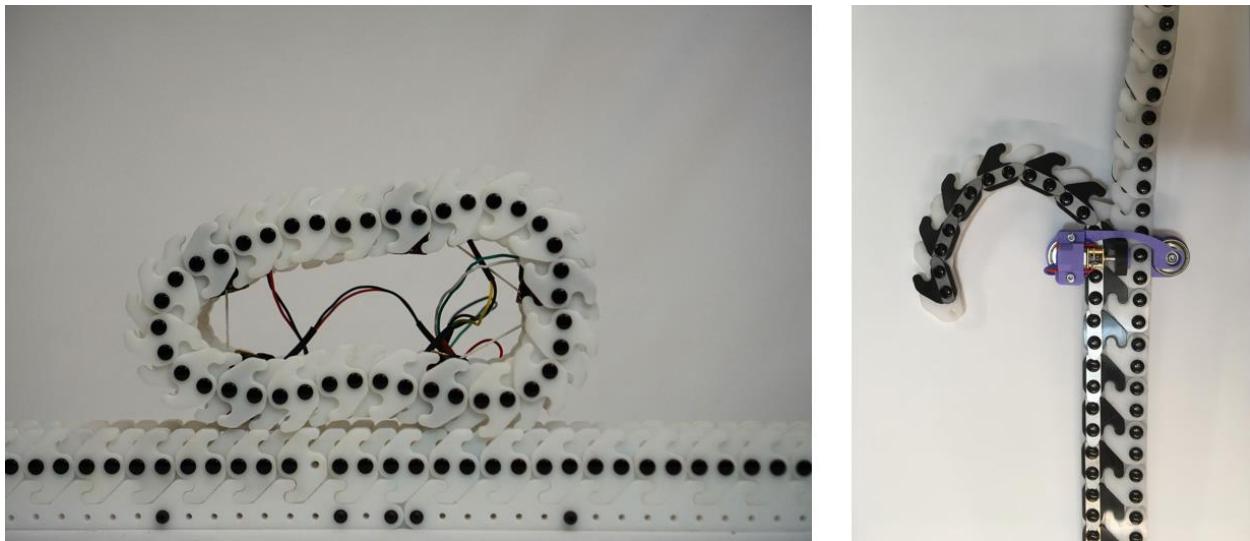
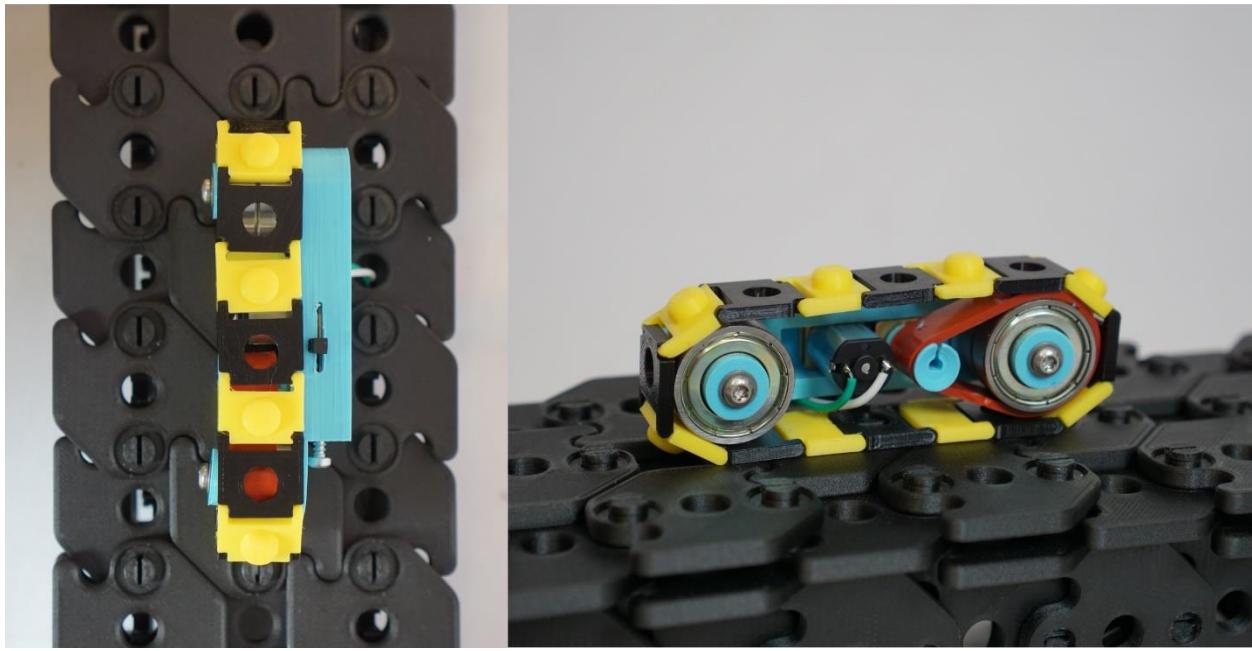


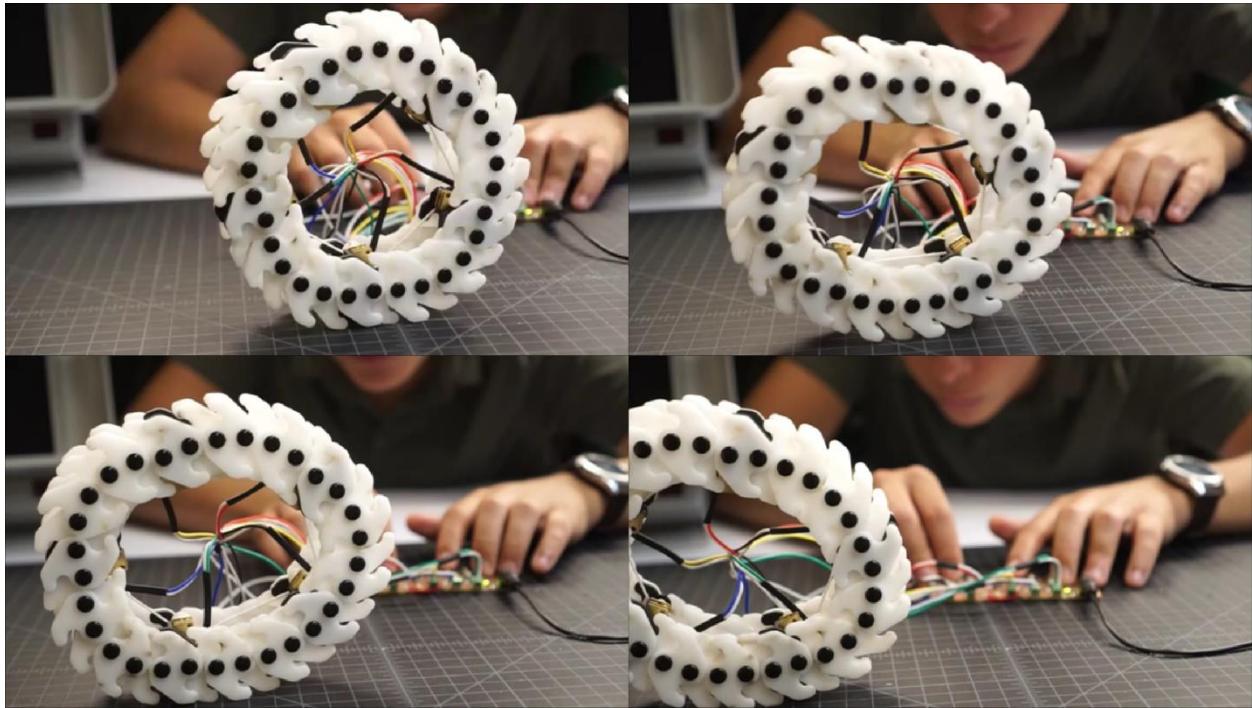
Figure 61 Robot constructed from Zipped and a robot that interacts with the Zipped system two zip two strands.

Initially, teeth need to be attached together to form strands. This can be accomplished via an assembly machine that picks up and moves parts snapping them together, such as a pick and place or traditional robot. This machine could be stationary or mobile, traveling along a strand or the surface of the structure. Teeth can be placed individually onto the surface of a structure. A prototype was fabricated to demonstrate the ability for a robot to locomote on the surface of a Zipped structure. The custom caterpillar tracks lock into the snaps and as they rotate the robot moves forward linearly. Two of these crawlers could carry the assembly machine described above.



*Figure 62 Picture of a prototype that can locomote on the strands.*

An alternative method for the parts to locomote on the structure was fabricated and is pictured below. Zipped pieces were assembled with N20 geared DC motors that actuated tension lines. This motorized ring can deform and roll along the leading edge of a Zipped strand, pictured in Figure 61. Additionally, this robot can traverse rough terrain as it morphs around obstacles. This actuated tendon design was also used to automate some of the mechanisms described previously, including an inchworm type robot that can fold and scoot, as shown in the videos linked in the Appendix.



*Figure 63 Flexural rolling robot*

In these proof of concept proto types, the robots were hand piloted, but work began on a network of distributed motor controllers which fit onto the Zipped pieces and could connect to a power tether running the length of the strand. These electronic boards need to be on the outside of a strand, as their uneven surface prevented them from being sandwiched by other parts, however, they do not prevent strands from Zipping.

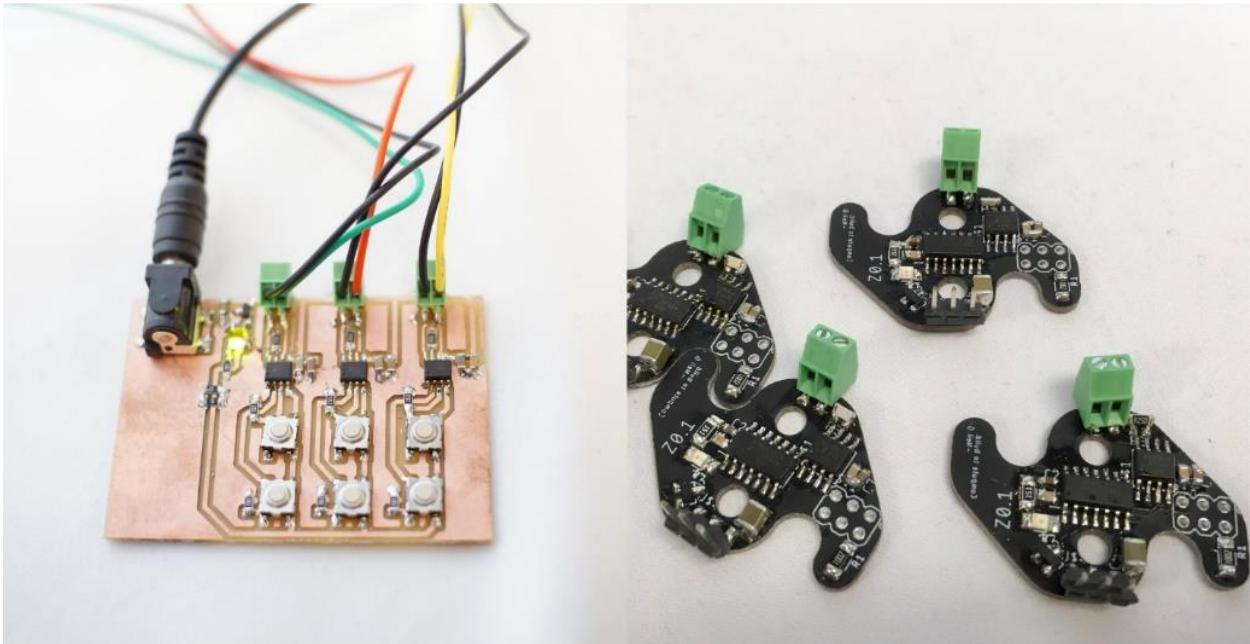
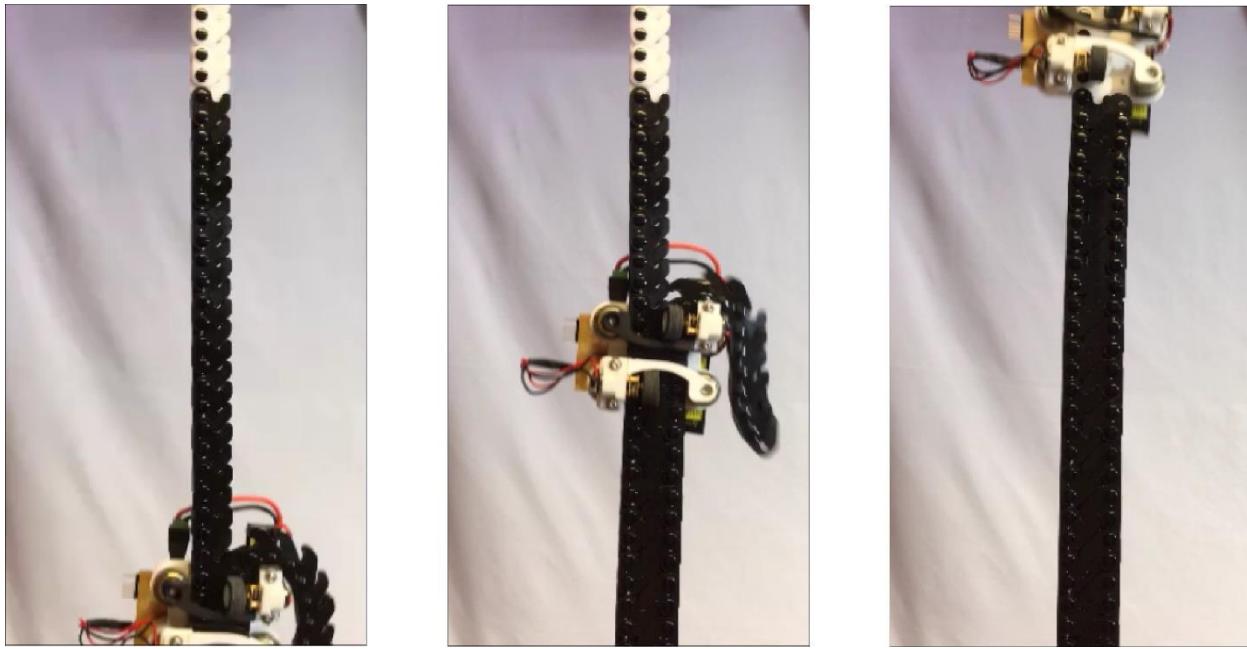


Figure 64 Prototype control boards (L) Button controlled used for flexural robots and mechanism tests. (R) Zipper shaped circuits that fit onto the pieces for distributed control.

It is believed by the author that an automated, flexural strand robot could control its motion allowing it to zip itself to a structure. This was not demonstrated however as it was found to be quite challenging. Alternatively, several robotic carriages were designed and fabricated that ride along the edge of a strand and zip strands together. One of these robots is pictured below, which pushes on the strands from all sides. Its geometry is custom to the geometry of the strands. In this instance the robot was designed to zip two single sided strands; it would not fit a single and a double-sided strand.



*Figure 65 Robotic carriage which zips two strands into a beam.*

Though a carriage robot is intuitive to design and build, it needs to be customized for each strand configuration. Below is a prototype for a carriage robot that connects several strands at once, including angled edges. This prototype was tested and was found to be functional but finicky. Fundamentally this approach is lacking as it requires a unique robot for each configuration of strand assembly. The robot body must also be customized to the part type, here no orthogonal pieces were used. One could imagine an alternative approach using the base, locomotion robots shown in Figure 62. These caterpillar tracks could be oriented in a spring-loaded U-shaped orientation that rides along the leading edge of the structure zipping strands to it. Though the robots initial placement on the structure, as well as strand feeding to the robot, still needs to be determined, it would allow construction of arbitrarily large structures.



*Figure 66 Robotic carriage for a unique T shaped strand configuration.*

Though several of the robots pictured allow parts locomotion and could be used to transport parts from one location to another, they are not designed for rapid transport or large loads. Thus, the following

prototype made from zipped pieces was fabricated. It is actuated by an internal gear moving the center of gravity, i.e. it is a hamster wheel. Parts can be added to the front and back of the wheel as the snaps are exposed. As the weight on one side of the wheel is counterbalanced by parts on the other side of the wheel this design can grow arbitrarily large in radius and the physics will only improve its motion as the larger the radius the more torque. Instead of this internal gear and motor, one could imagine a robot such as the one pictured in Figure 62, locomoting on the outside of the strands to drive the motion of the wheel.



*Figure 67 Hamster wheel robot constructed from Zipped pieces which can self-locomote*

As the strands themselves can form looped chains it is of interest to see if the robot in Figure 62 and the fabrication machine described could be constructed from Zipped pieces, thus the following model was made. Here the tracks on the outside are constrained by pieces sandwiching them but can rotate freely. Though this is not a mechanized prototype, the motion and machine structure prototyped here demonstrates potential for the system to fabricate load carrying vehicles.



*Figure 68 Prototype toy vehicle whose tracks can rotate freely.*

## 6.1 CASE STUDY: THE BRIDGE

Though the use cases for zipped spans much of infrastructure, bridge construction was of particular interest due to the Army Research Labs vision. Several designs were made to evaluate how a bridge could be assembled using the Zipped system accompanied by hand-built prototypes. This study aimed to demonstrate the steps that would be required to construct a bridge as well as demonstrate potential fabricable designs. Previous studies have shown bridge design can be optimized for different scenarios, particularly the length of bridge required [34]. These designs were all made to cross a 1m gap using the 1.5"x1.5" pieces with snaps.

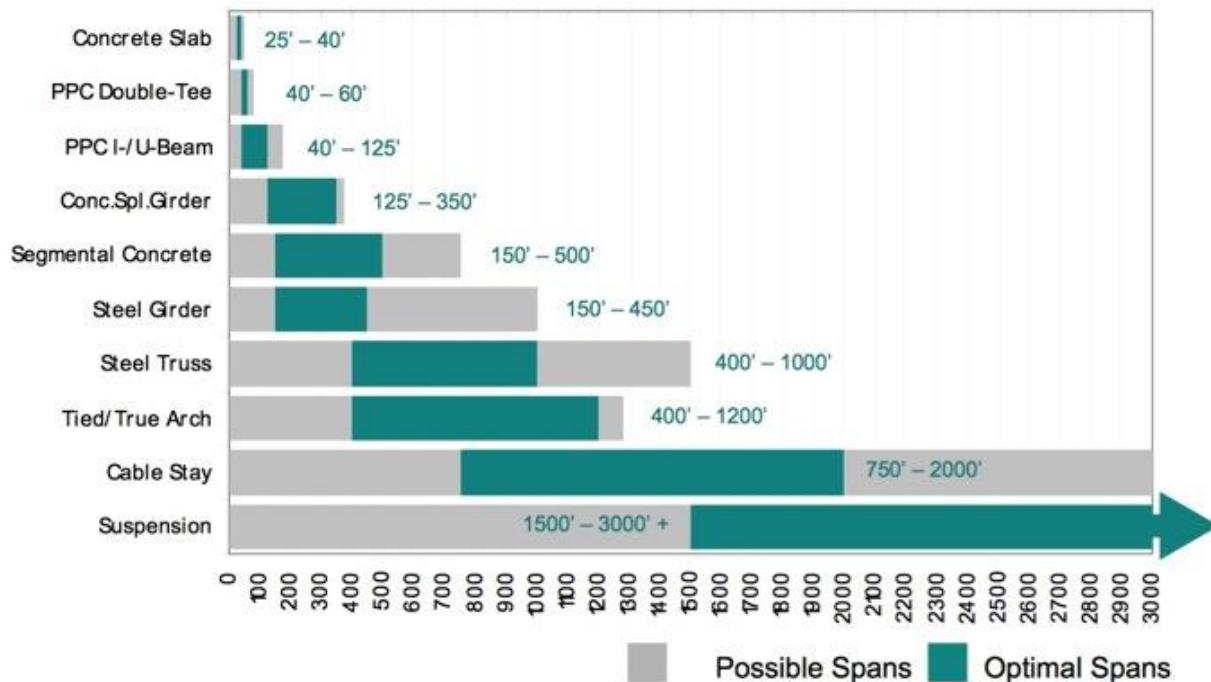


Figure 69 Bridge types and optimal span lengths in feet. Source: Portland-Milwaukie Light Rail Project [34]

For short distances a beam bridge is typically the most economical. The first thing to consider when spanning a gap is creating a secure foundation to extend a cantilever from. In the pictures below, the Zipped strands extend to the left counterbalancing the cantilevered section until the entire gap has been spanned. The second image shows two strands zipping to form the initial cantilever. Before they are zipped the strands are flexible and can be moved into position. The third image shows a strand with side snaps uncurling on to the structure. Strands are added to form a beam bridge that spans the entire gap. Additional strands can be added to extend the width of the bridge or increase its rigidity. When no longer needed the bridge is deconstructed moving the parts from the left to the right to adjust the center of gravity, making the foundation on the other side of the gap.

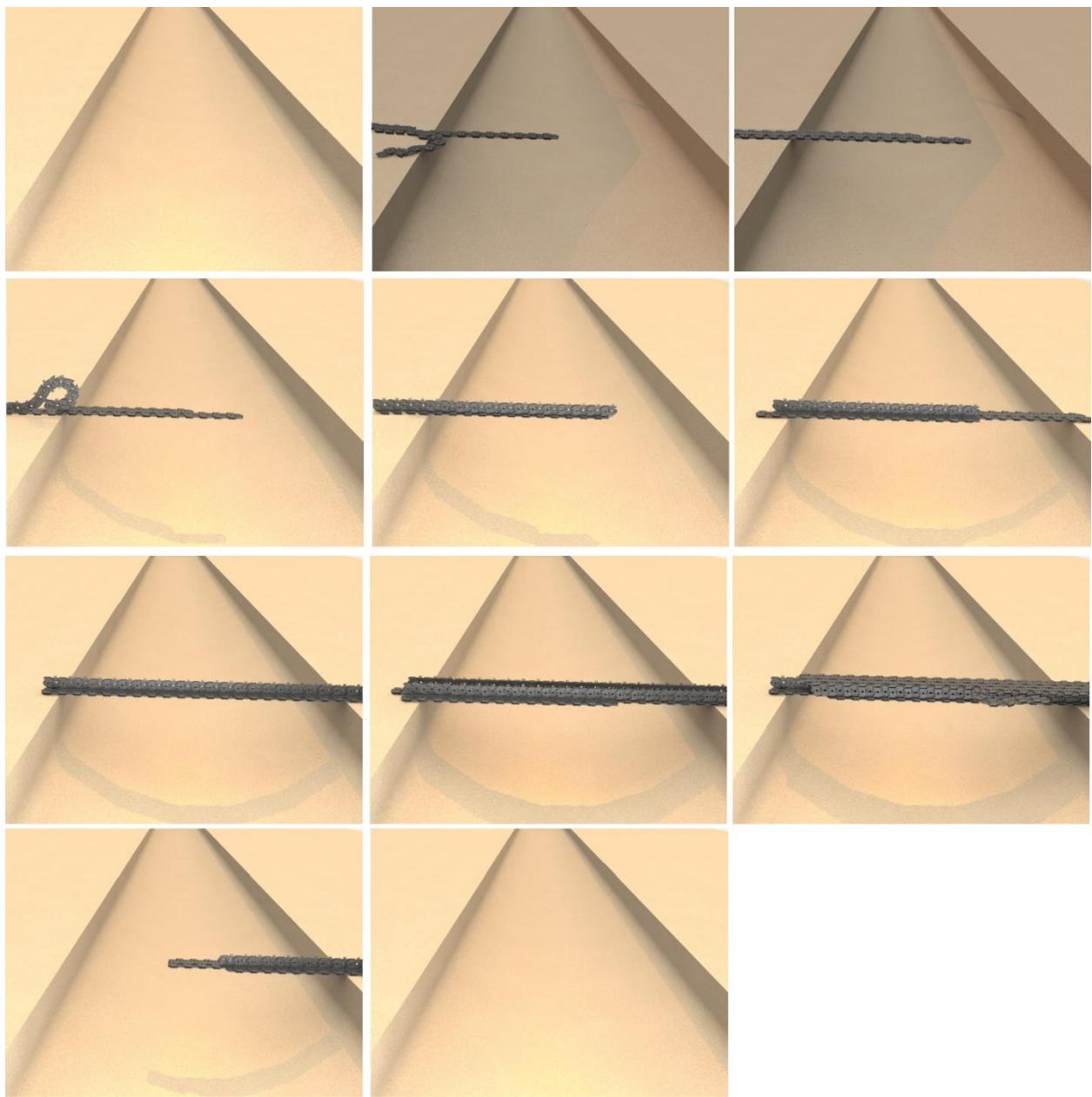


Figure 70 Construction stages of a zipped beam bridge spanning a 1m gap.

There are many geometries for beam bridges that can be constructed using Zipped, however, these bridge designs are fundamentally what has been described previously as a “channel”. To strengthen the design more strands can be used to increase the second polar moment of inertia. Orthogonal locked strands can be used to reinforce the profile against shear and torsional forces. Below a simple beam bridge is shown, which was built by hand.



Figure 71 Hand-built beam bridge spanning a 1m gap.

For longer bridges an arch style may be more advantageous. Pictured below is a rendering of a design that uses 2025 double sided parts and 180 double sided parts with side snaps. A similar design was constructed by hand. Though this design has a front and a back face connected by strands with side snaps, it could also be fabricated from very thickly stacked strands. If the bridge were monolithically designed from tickly stacked strands it would be roughly 6 times denser, though a combinedly of thick stacked strands and cross bracing side snap pieces could also be utilized. Using either method it is possible to tile the design making it indefinitely wide.

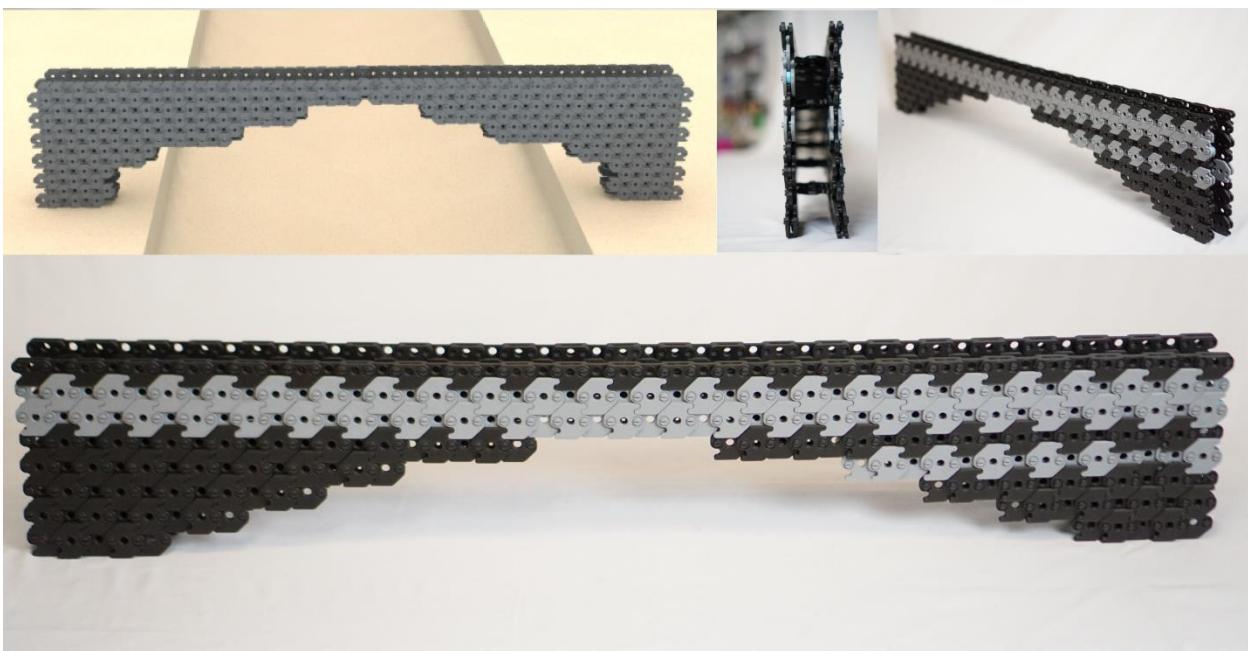
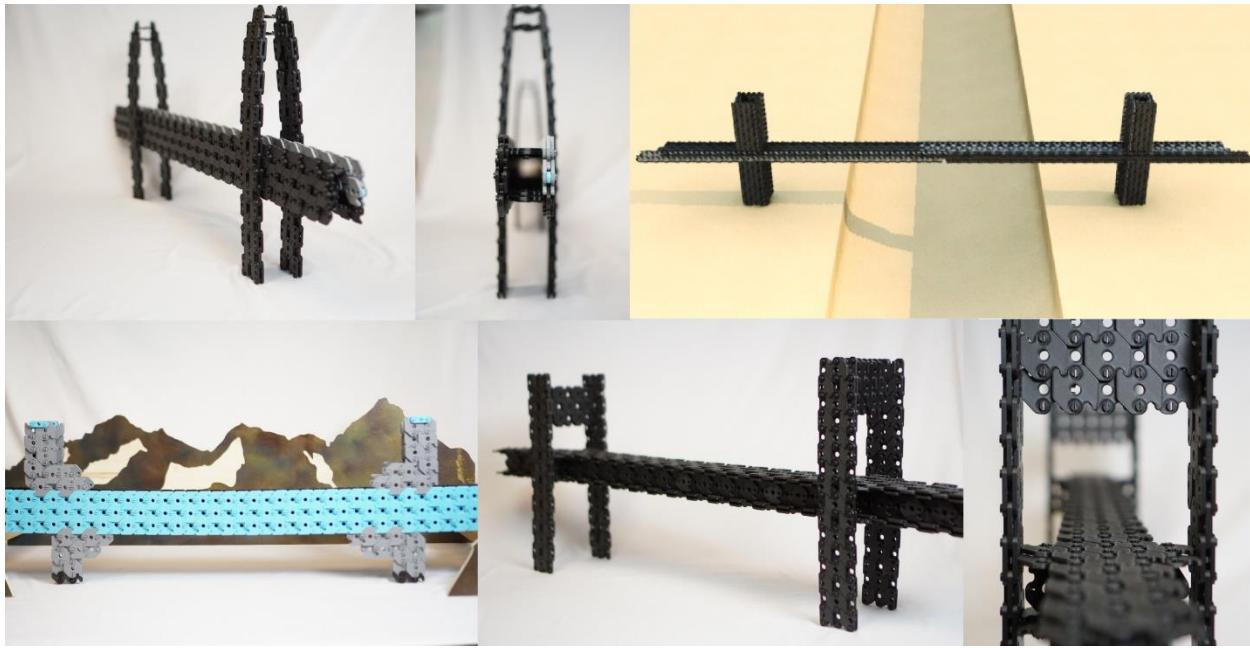


Figure 72 Arch bridge design showing rendering and hand-built versions shown from the front, side and at angle.

As studies show suspension bridges can span the largest gap of any design, it is desirable to prove this design concept. Below a design for a suspension bridge rendered using 9888 double sided parts and 880 double sided parts with side snaps. Additionally, several hand-built suspension bridge designs are shown. Each design has two towers and a road bed which is lifted off the ground. Cables could be connected between these towers and the road bed to increase its strength and supported length, allowing the road bed to be extended.



*Figure 73 Suspension bridge design.*

Notice the towers in the designs above are each connected differently. The first uses the face snaps to lock zipped strands orthogonally. The rendered design pulls strands from the vertical channels and branches them to form the road bed. The lower left re-uses the beam bridge shown previously and adds towers by locking two strands in line with the road bed, then turning them vertically. Finally, two images on the bottom right show how side snaps were used to connect vertical sheets. This shows the diversity of fabricable construction options, as well as creativity in the design space, which Zipped offers.

## 7 CONCLUSION

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The Zipped system is an approach to assembling structures that mirrors biology as 0-dimensional parts are assembled into 1-dimensional strands, which zip together into 2-dimensional beams with additional strands branching, bending and merging to form 3-dimensional structures. When un-zipped, strands are very flexible. However, when zipped together, the strands lock to form rigid segments that passively hold position. They can be un-zipped easily by a directed force. This allows partial stiffness and flexibility to be designed into structures or mechanisms using the same few parts.

Zipped parts were prototyped at several different scales and with different materials. It was shown they can be affordably, mass manufactured using common processes. Several thousand were produced in lab for prototyping, from which their ability to create a variety of forms was demonstrated. From these prototyped parts the bulk material properties were found. Zipped sheets were shown to be anisotropic, in some orientations almost matching the raw materials tensile stress at yield, in other orientations it performed as poorly as 1/60<sup>th</sup>. It was shown however that un-zipping and re-zipping has no effect on the material and it was demonstrated that increased stiffness can be designed into beams by architecting their geometry and the parts orientation. Thus, with careful consideration to the materials orientation structures with properties nearly matching the original part material could be fabricated via this assembly method.

Additional work is needed to tailor this system to any particular use case. Though initial prototypes for automated assembly were demonstrated, significant additional work is needed to robotically construct structures. The size, setting and environment the structure is in defines the robots' functional requirements. For instance, the zipping robot needs to drag a length of un-zipped strand to zip it in place. Depending on what the teeth are made of this could be a significant load, perhaps additional robots are needed to help carry it. The robot likely could not rely on fiction wheels to drive as were used in the small scale, proof of concept demonstrations. Additionally, significant work on robot architecture is needed as no robot was designed to address the assembly of strands, addition of parts onto a zipped surface, adding or removing rivets, zipping branched geometries, attaching side snipped planes and numerous other required functions. Ideally, a single robot constructed from Zipped pieces could serve all these functions. The robots and mechanisms demonstrated lay the ground work for believing Zipped a viable architecture for assembly of flexible, modular robots by incorporating actuated elements.

Before constructing larger, or more complex systems a design tool is critical. CADing the bridge renderings shown in this thesis reaches the limits of SolidWorks and my laptop. As part types fed into strands limit what is fabricable by those strands, it is imperative that each step of the fabrication process comply with the overall structural design. The design tool could be a top-down design approach where the structure desired is designed as a bounding geometry, then divided into fabricable segments. This approach would aim to re-create the designed structure out of Zipped pieces as accurately as possible. Alternatively, the design tool could be based off structural primitives that can be assembled by Zipped and limit the design space to only what is constructible. Either way, ensuring fabricability during the first stages of assembly is imperative for scaling.

Additionally, the desire to robotically construct architectures necessitates a system to convert the design into robot commands. For ease of design most of the demonstrations in this thesis were constructed from a single part type, either the orthogonal locking pieces or the double-sided tooth geometry. Multiple part types allow tailoring of the structural properties for enhanced stiffness and strength, and with a streamlined design-to-automation pipeline the complexity of having additional part types should not be a barrier. Though it is a more complex design space, as a greater diversity of structures emerges, the fabrication process should not be significantly altered.

Mirroring how molecular biology assembles into chains that zip and fold into structures, this fabrication process is unique among human made assembly set. Zipped, is a versatile building block, that is designed and characterized in this thesis. The demonstration presented here are a start towards a sustainable, scalable future of assembly.

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## 9 APPENDIX

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Videos: [https://gitlab.cba.mit.edu/falcone/structural\\_robots/wikis/home](https://gitlab.cba.mit.edu/falcone/structural_robots/wikis/home)



Figure 74 Concept for a potential structure utilizing the play between the parts to form a curved canopy.



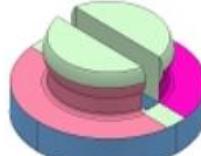
Figure 75 Concept for a lattice structure made using Zipped

Rivets or pins are required for use with 2-dimensionally fabricated pieces. The most affordable plastic push-in rivets were found for US\$0.12 at AlliedElec. Sourcing rivets long enough to fit at least 3 pieces is a significant limiting factor to fabrication. Most of the rivets purchased for this thesis came from McMaster Push-In Rivets and cost \$12.24/50 which is USD 0.24/per. They are also found in bulk from Mouser, however they're still USD 0.226/per at quantity of 5,000.

*Table 3 Quotes to laser cut these parts at scale from Yubo MFG. [26]*

Part	Image	Price per	Total
Single sided tooth		US\$0.72 ea x 3000pcs	US\$2160
Double sided tooth		US\$0.75 ea x 3000pcs	US\$2250

*Table 4 Molding quotes from Protolabs*

Part	Image	Tooling cost	Price per unit
Base backer		\$1,760	\$2.66
full pin		\$4,900	\$2.52
half pin		\$1,695	\$2.51
Overmolded assembly		\$2,495	\$3.22
Pin substrate		\$4,915	\$3.07

## 9.1 MECHANICAL ANALYSIS APPENDIX

Repeated tension was applied to determine the snaps strength and durability. This experiment was done using Onyx material on a MarkForged printer and on PLA prints with a Prusa. Results were calculated by averaging the maximum tensile force supported on each run per material. These finding show Onyx to be more repeatable, thus the Onyx parts were used for further experiments.

*PLA averaged maximum load  $49.8 \pm 11.7\text{N}$ , Onyx averaged maximum load  $20.2 \pm 2.2\text{N}$*

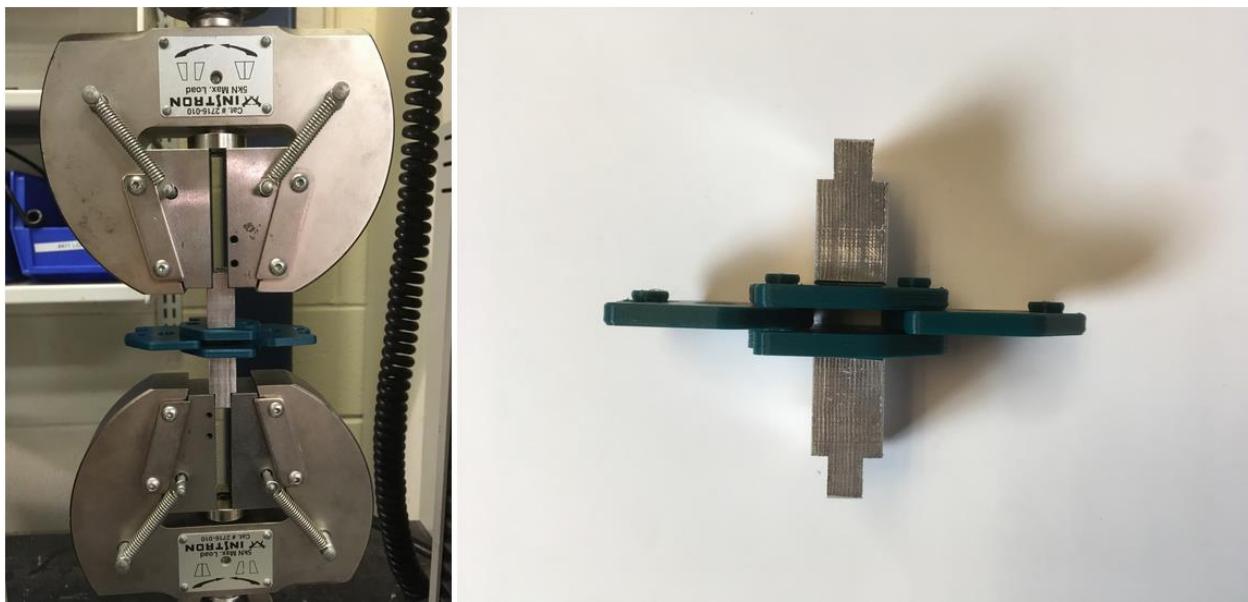
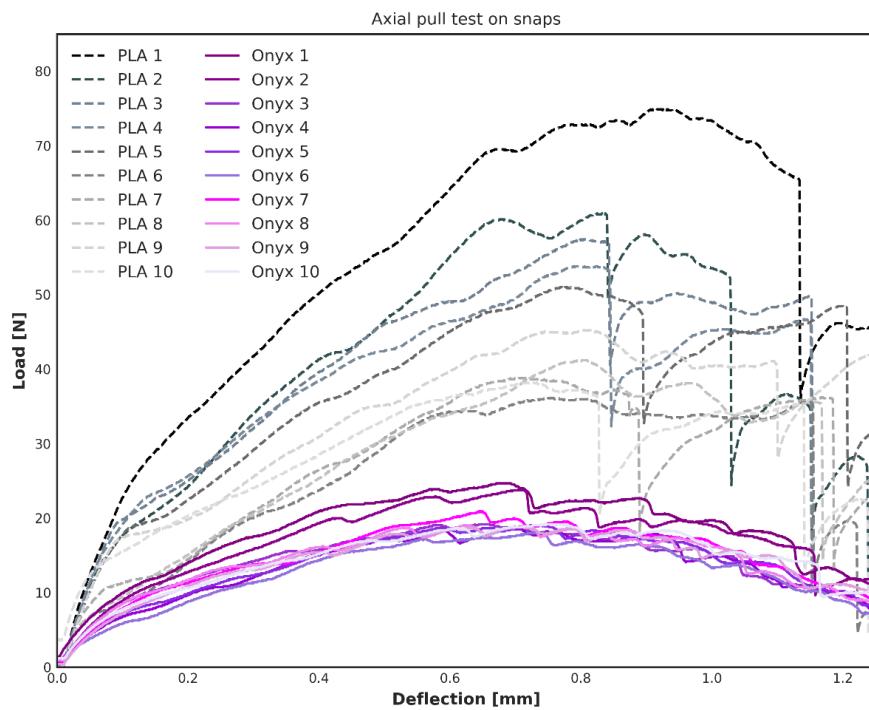
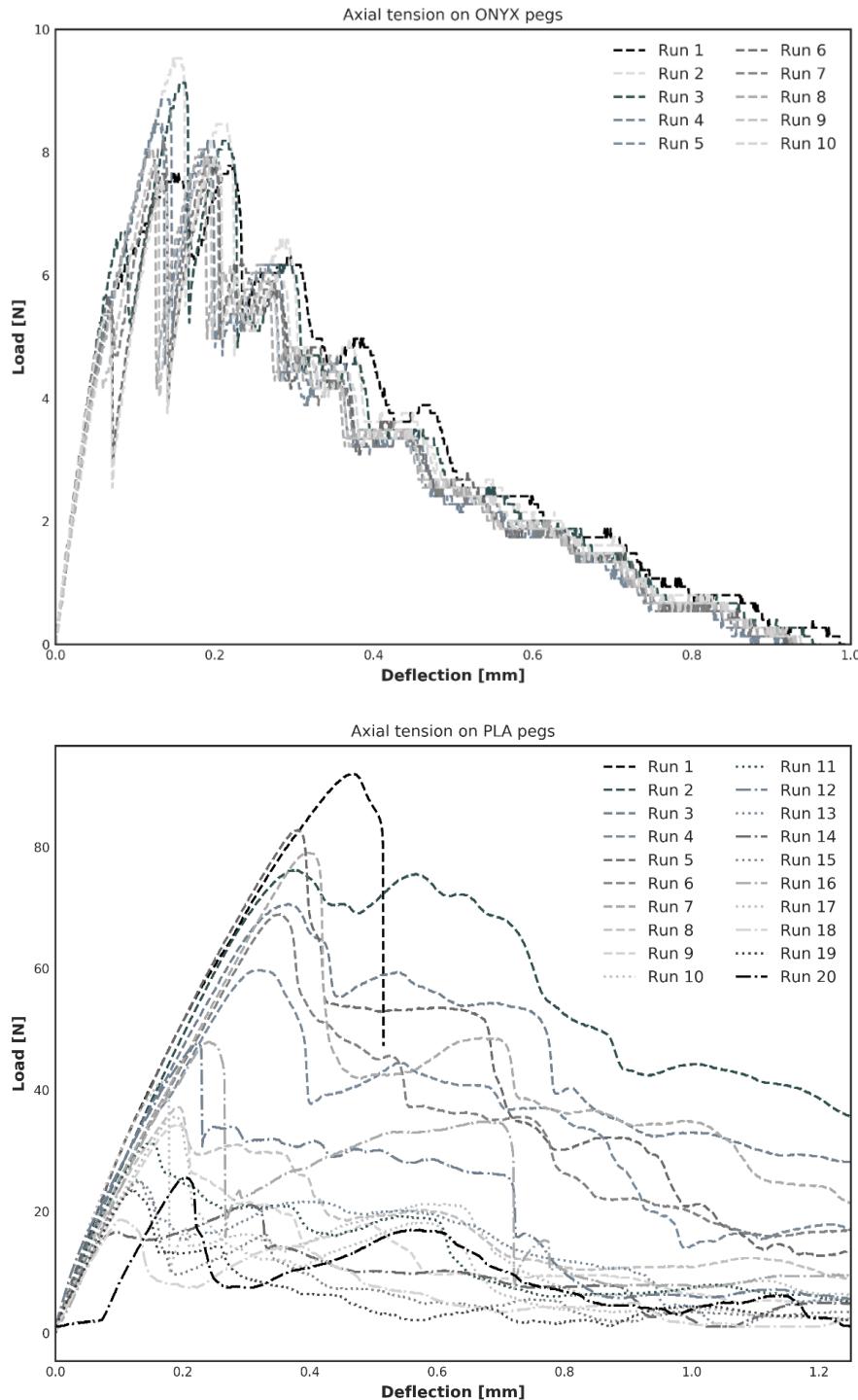


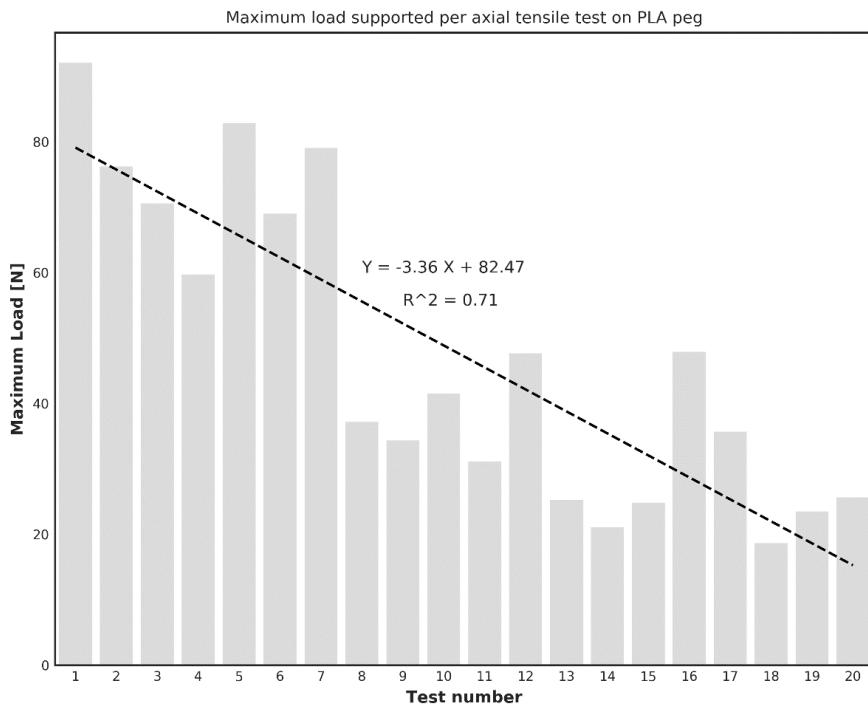
Figure 76 Experimental set up for axial pull test on snaps and press-fit pets inter-teeth connectors



Axial pull tests were performed with the same experimental set up as those above, using printed parts on Onyx and PLA with pegs instead of snaps. The same sample was tested multiple times, i.e. Run 3 is the third time the parts were pulled apart, etc.



As it appears from the above charts, the maximum tension supported by PLA pegs varies significantly more than that for the Onyx samples. These maximum values were plotted against sample number to show the negative correlation.



Empirically, the side snaps work very well when printed from Onyx. Printed from PLA they regularly snap. Prints were made with 100% and 15% infill using PLA on a Prusa machine. Samples with 15% infill seems springier and less likely to break, though this makes sense as less material should be more flexible, no tests of statistical significance were performed. Samples of Onyx (using MarkForged Eiger slicer pre-sets) and 15% infill PLA were used in a tensile test, as shown below.



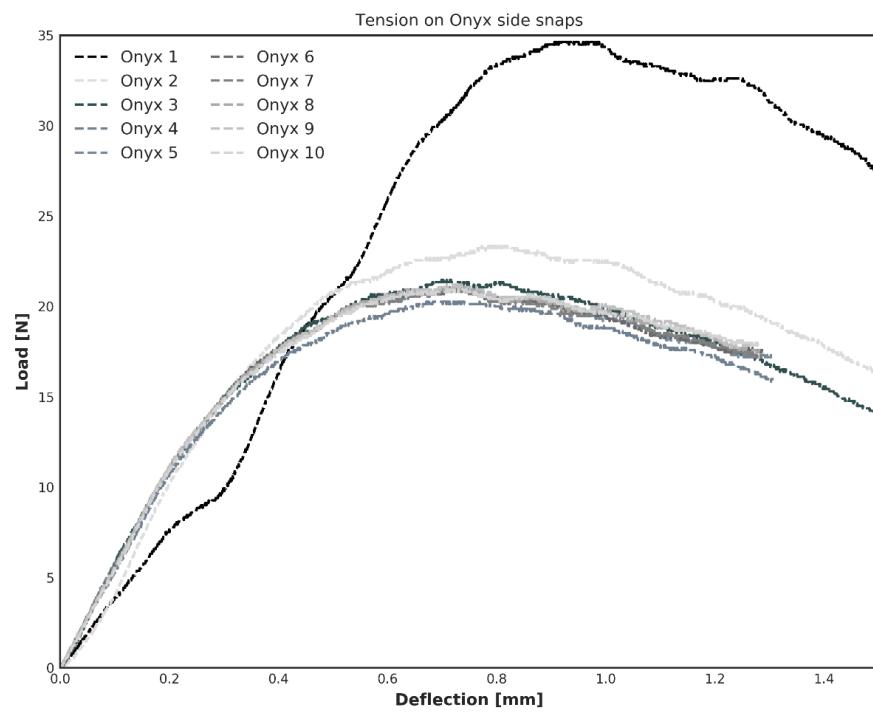
Figure 77 Experimental set up for side snap tensile tests.

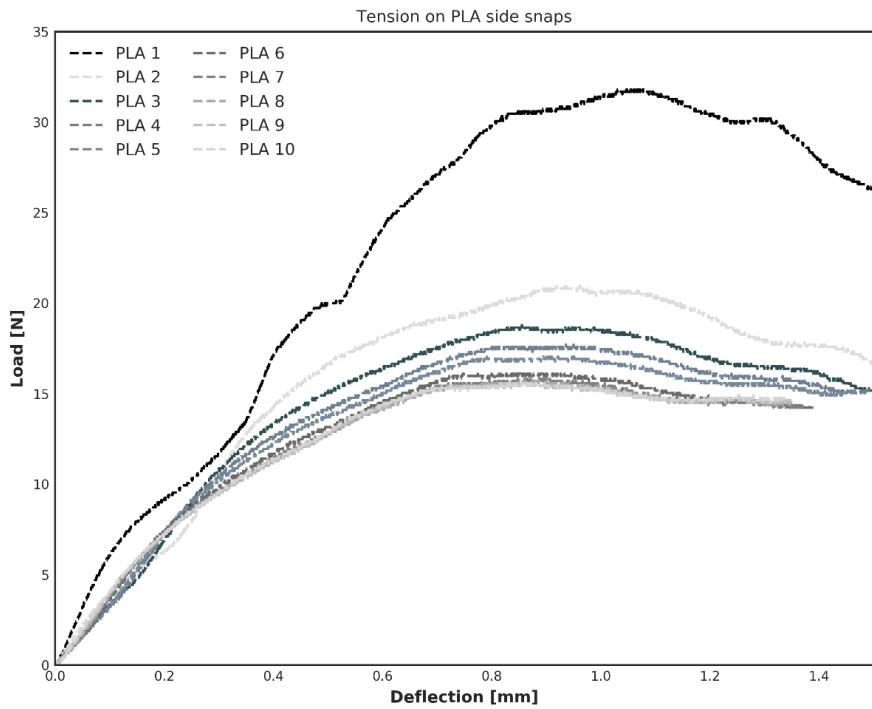
Instron tests were performed for both Onyx and PLA samples. The fixture for these tests is an exact replica of the hole the side snaps mate into, fastened to a block which can be clamped by the tensile

gripping jaws on the Instron machine. Results shown below demonstrate that the first time a side snap is used it is stronger than the following runs, indicating some plastic deformation in both PLA and Onyx.



Figure 78 PLA failure during side snap tensile test. Notice the peg is permanently bent inward.





A tension is applied to pull along the length of a strand loading the pins in shear. Both snap pins and peg pins are tested made from Onyx and PLA. A single sample for each design and material was made and multiple tests were run on each sample. The PLA snap test was only preformed once however as the snap showed visible damage. Also note, the PLA and Onyx samples are displayed with different scales.

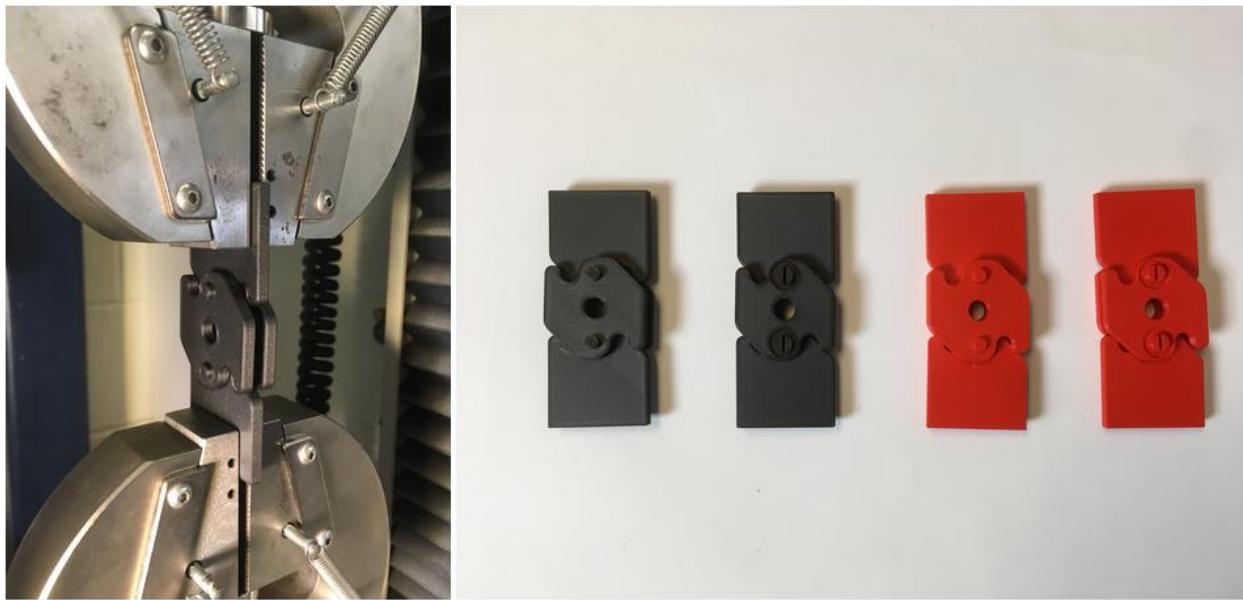
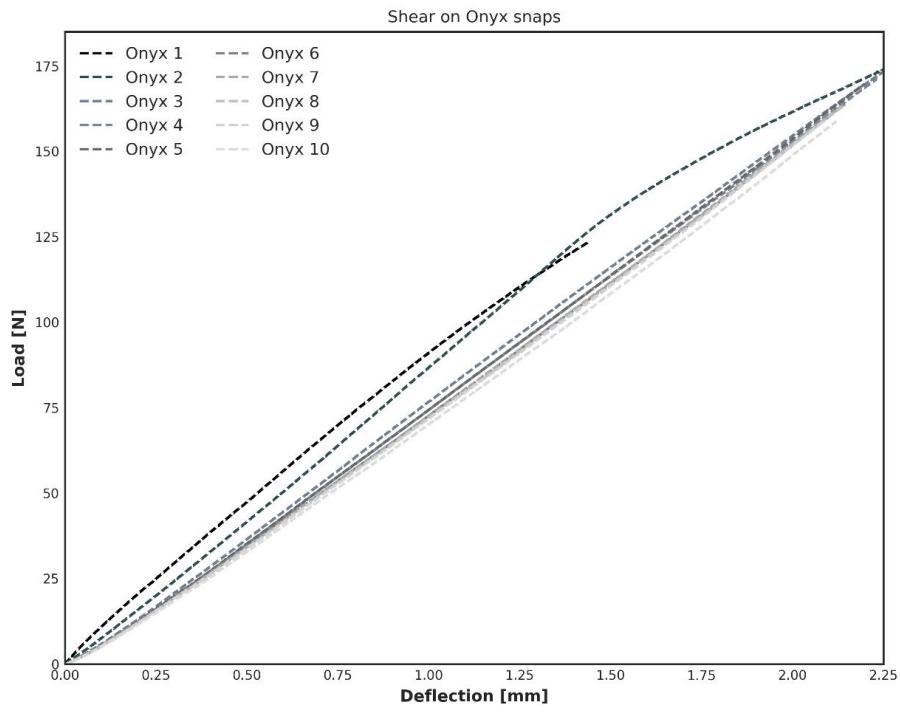
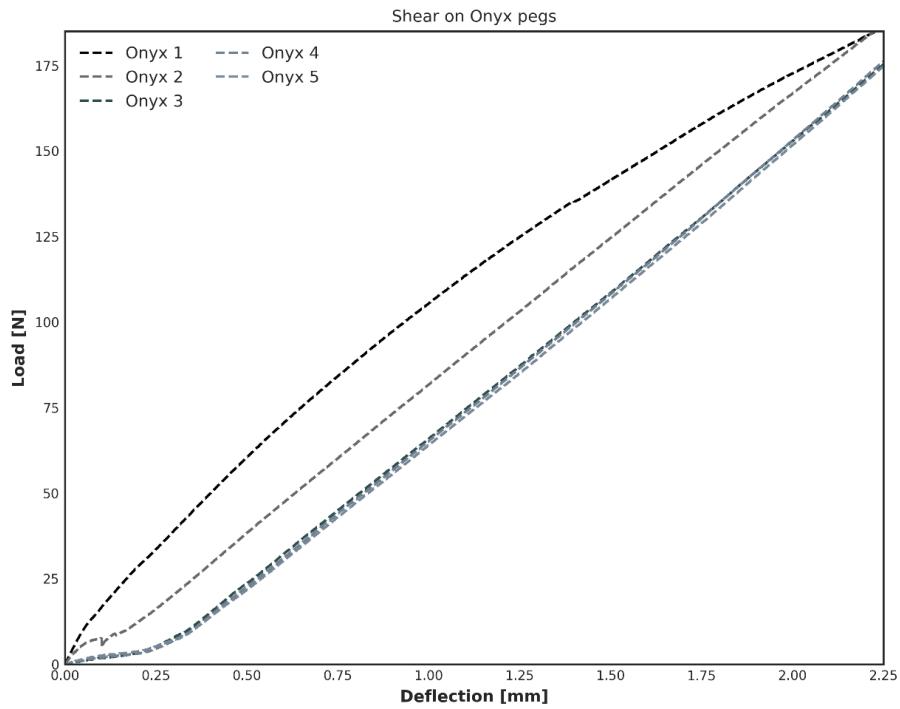


Figure 79 Experimental set up for shear on inter-teeth connections



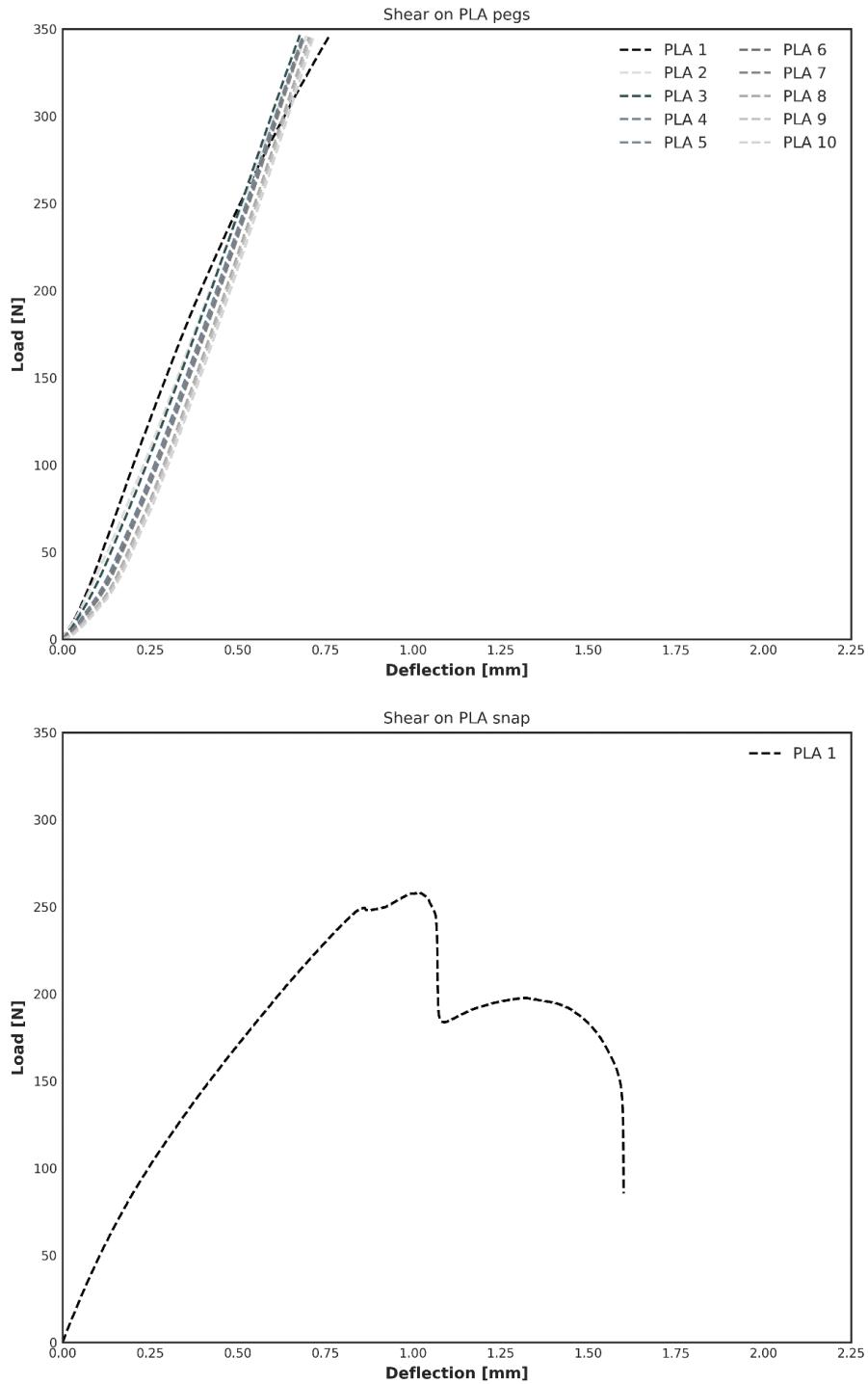


Figure 80 Note: Only one test was performed as the PLA snap broke during the first run.

Initial experiments were performed to determine the effect of cross bracing using side snap parts as well as pinned strings that act as shear webbing. 12" beams were tested in 3-point bending as shown in the figure below. Results show minimal variation between samples, which is not surprising as the cross bracing is meant to reinforce the beam in shear, a force the 3-point bending test did not apply.

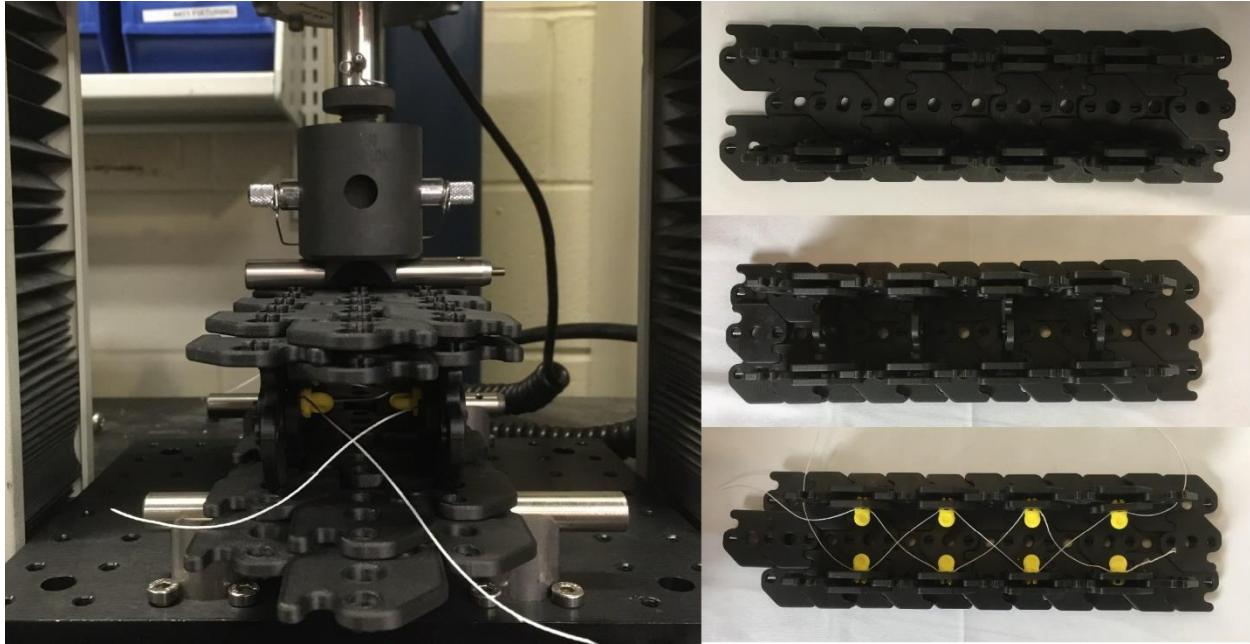


Figure 81 (L) Experimental set-up for channel comparison (R) Top shows the hollow channel sample (run 1-5), middle sample has locked tension cross pieces (run 6-10), bottom sample has string for shear webbing (run 11-15).

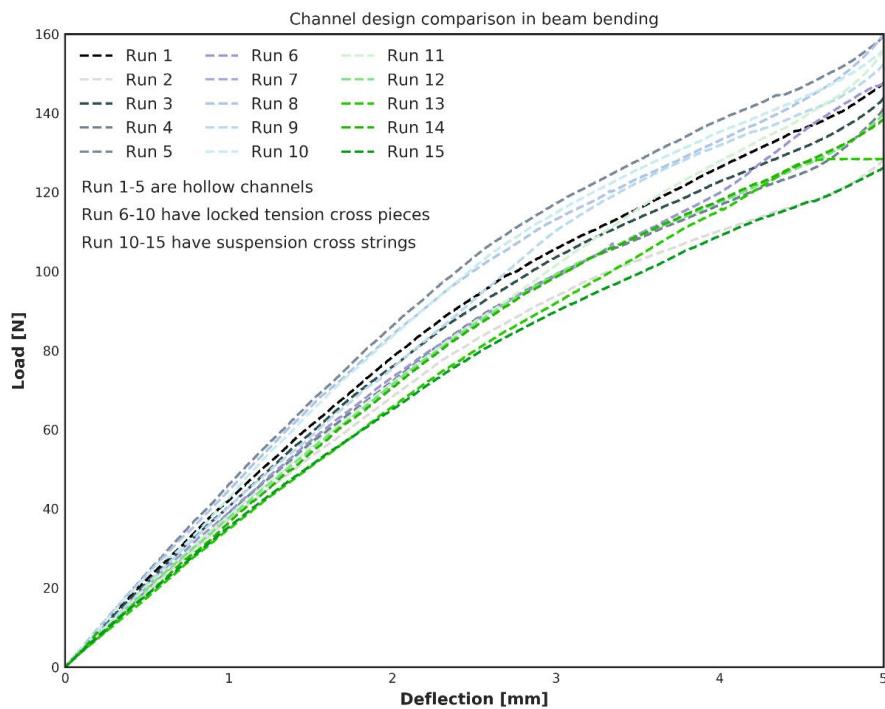


Figure 82 Results from experimental testing of hollow channels, cross braced channel and channels with tension elements