

Reliability is a Fungible Resource



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Fungible Resources

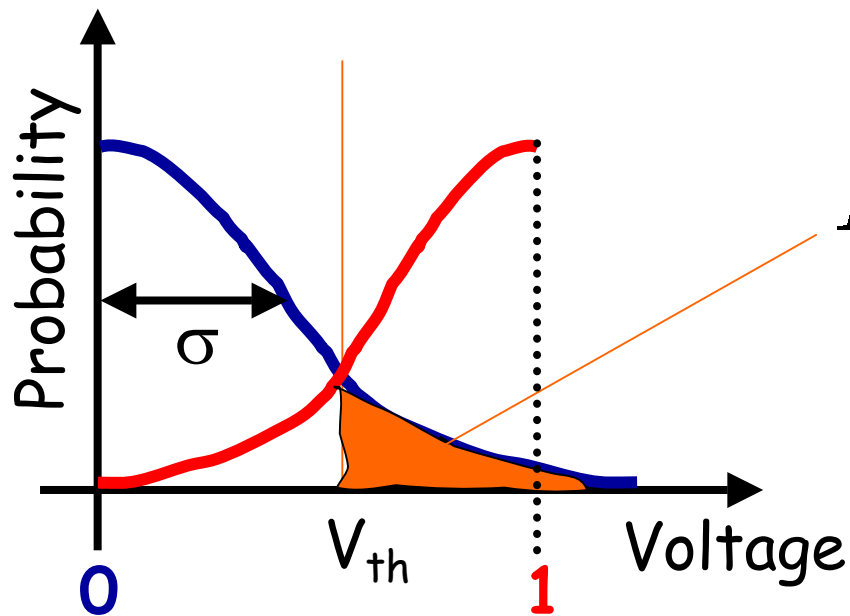
- \$\$ \Leftrightarrow Gold
- Euros \Leftrightarrow \$\$
- Mass \Leftrightarrow Energy
- Space \Leftrightarrow Time \Leftrightarrow Energy

- Claim:

Reliability \Leftrightarrow Space / Time / Energy

Reliability and Area (I)

- But also $P_{\text{fail}}(A)$
- Example: MOSFET

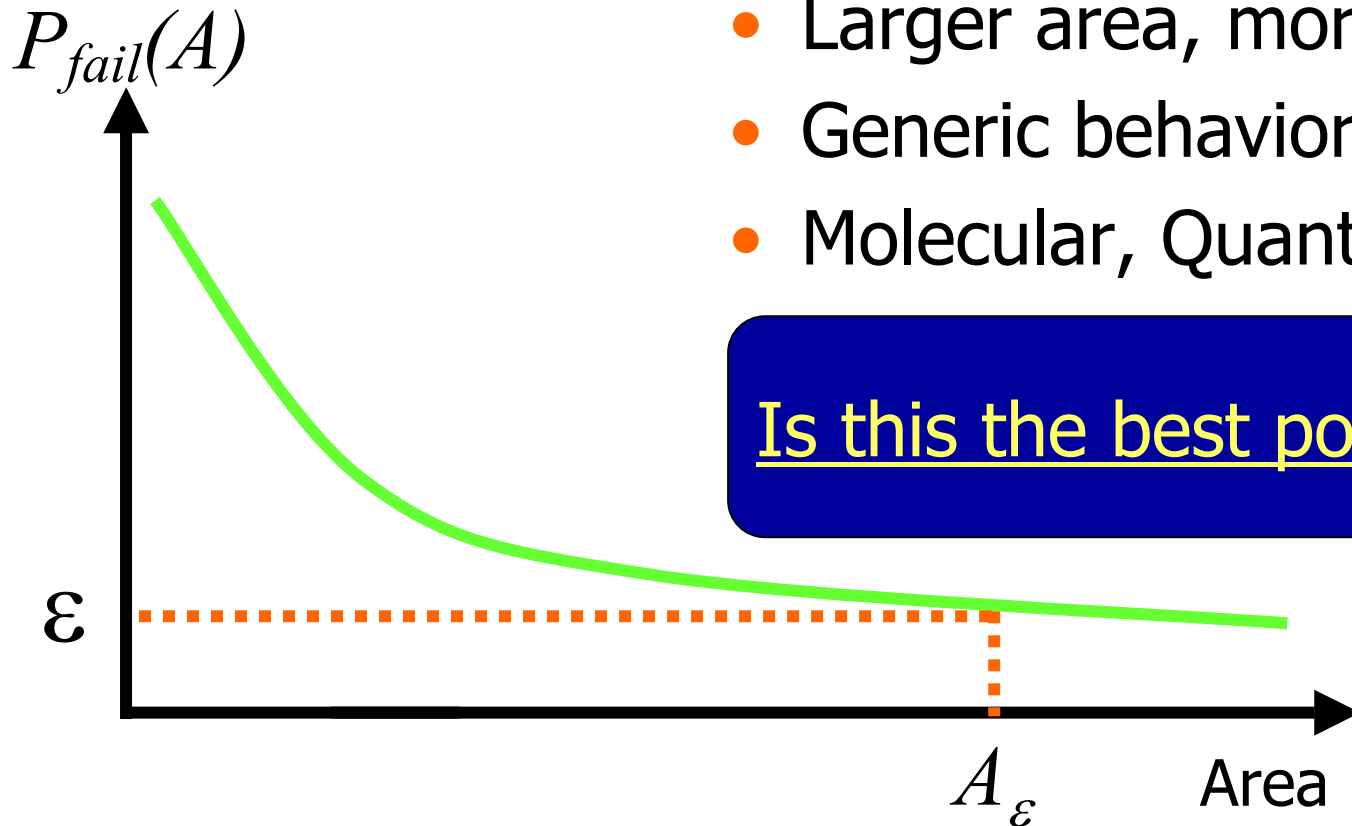


$$\begin{aligned} P_{\text{fail}} &= P[V > V_{th}] \\ &= \text{erfc}(V/\sigma) \\ &= \text{erfc}(A/A_0) \end{aligned}$$

$$\sigma \sim \frac{dV}{dQ} \sim \frac{1}{C} \sim \frac{1}{A}$$

$P_{fail} \leftrightarrow$ Area Fungibility

- What tradeoffs are possible?

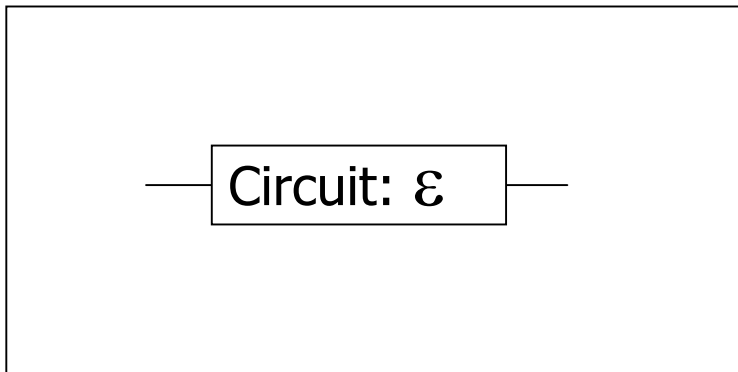


- Larger area, more reliable
- Generic behavior!
- Molecular, Quantum, ...

Is this the best possible?

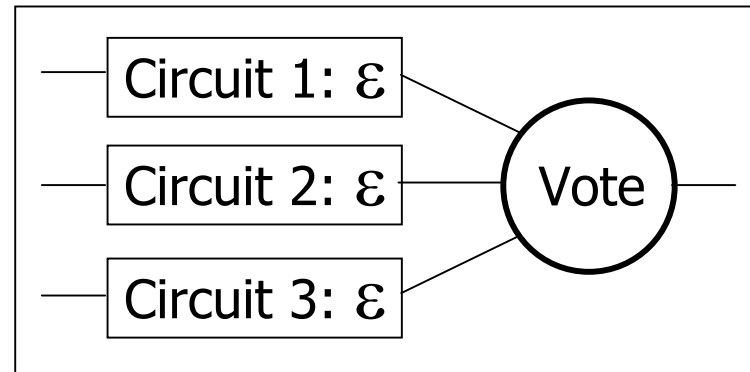
Reliability and Area (II)

- Consider a circuit which fails with probability ε



Area: A

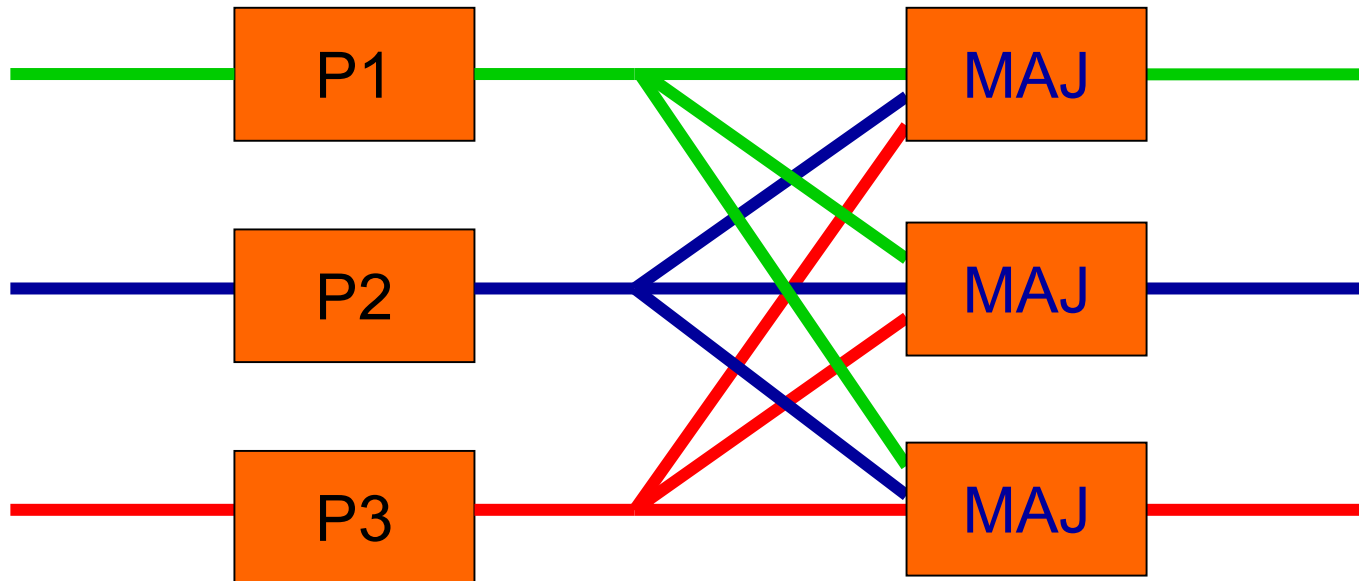
$P_{\text{fail}}: \varepsilon$



Area: $\sim 3A$

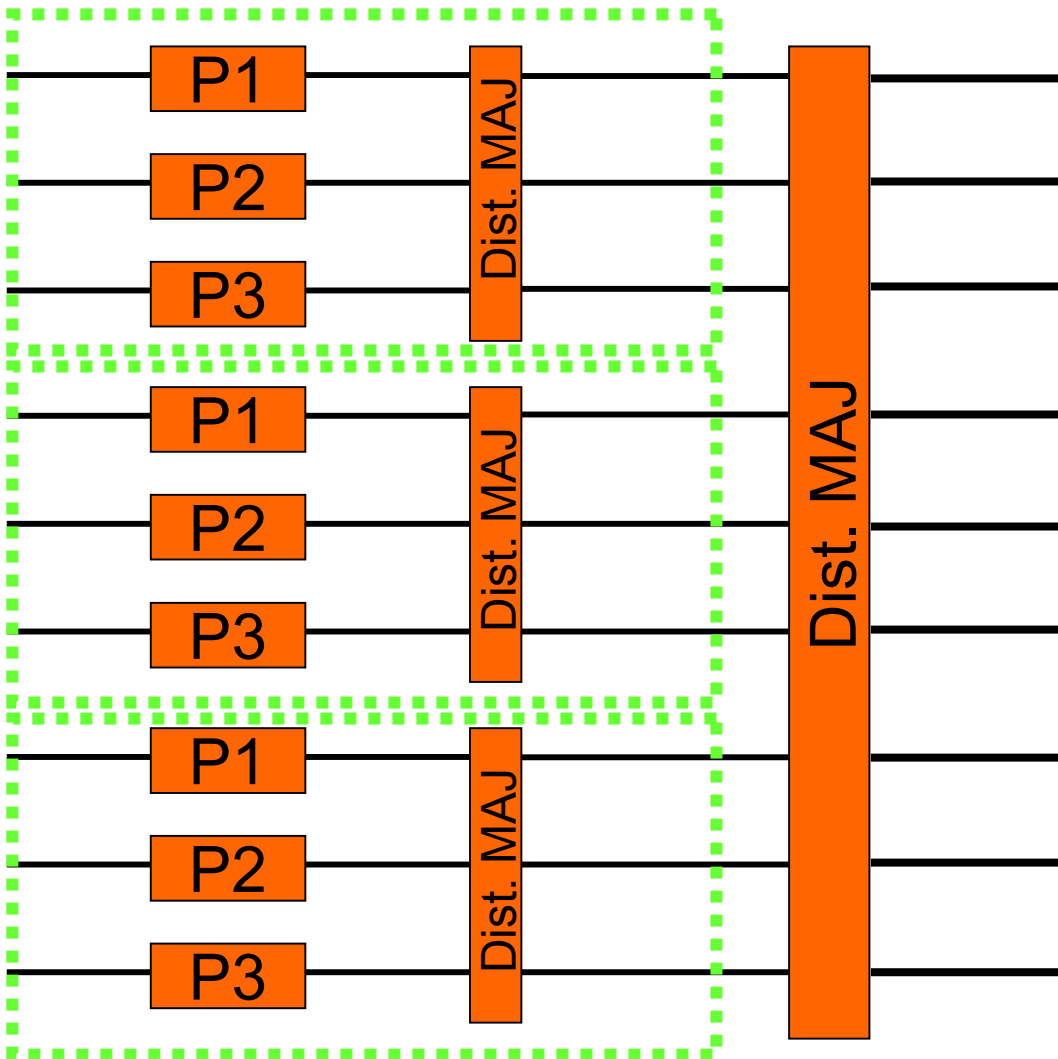
$P_{\text{fail}}: \sim 3\varepsilon^2$

Distributed Voting



- Immune to any single gate failure
- Failure probability: $\leq (cP)^2$

Double Encoding



Area: 3×3^2

Failure probability:

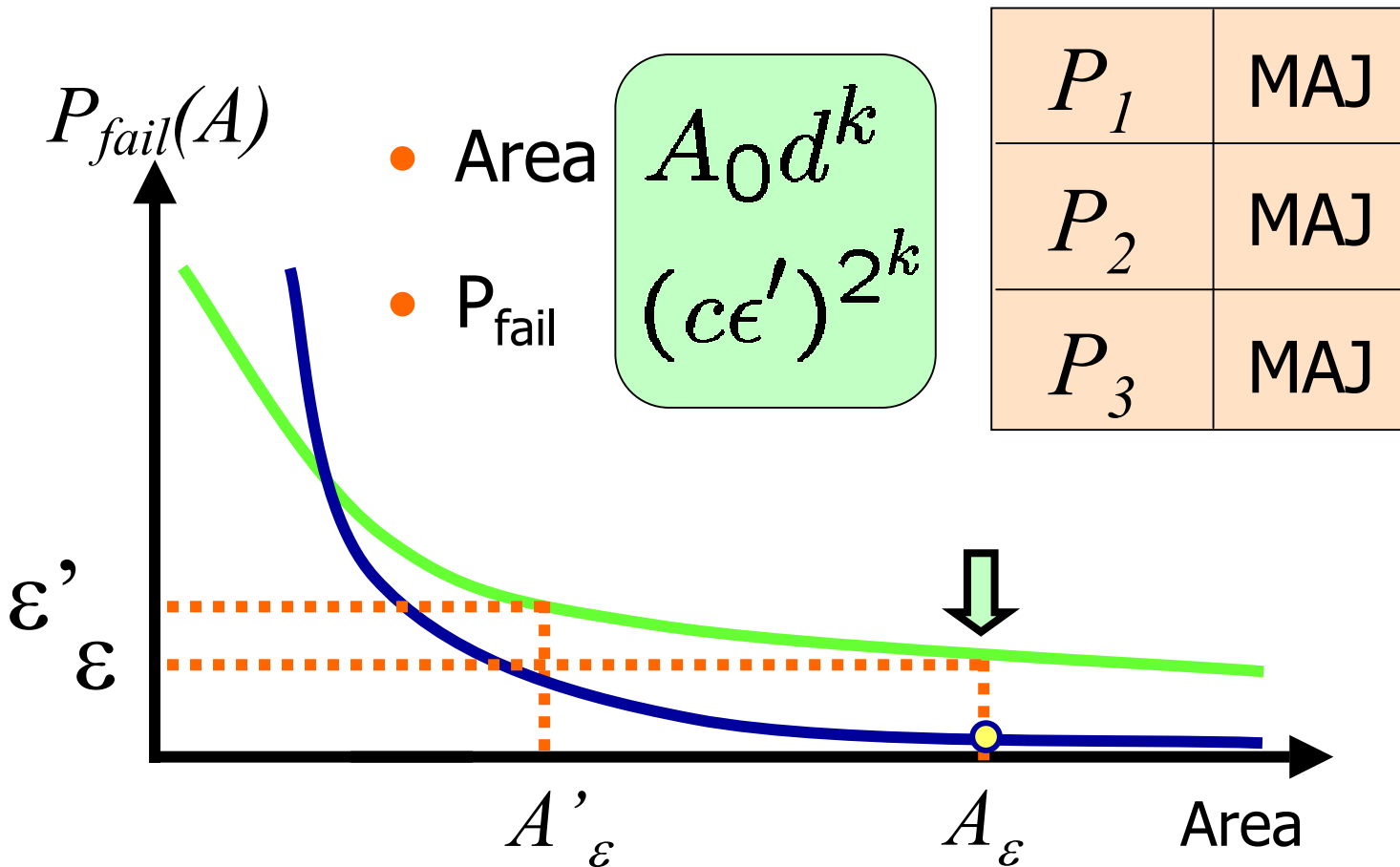
$$F \leq (cP)^4$$

Recursive Encoding

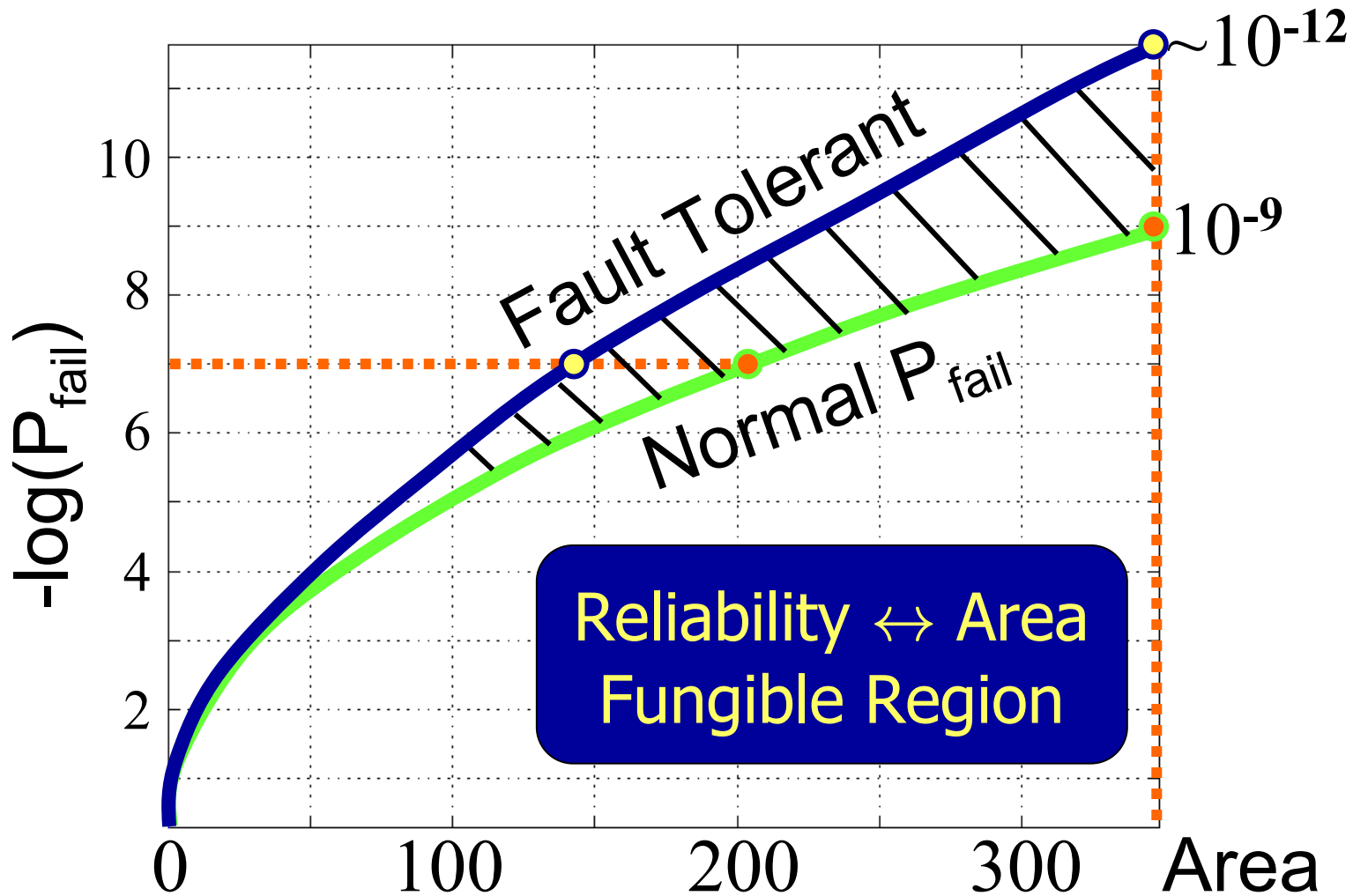
- Recursion: k
- Area: $A_0 d^k$
- Failure probability: $F \leq (cP)^{2^k}$
- Threshold: $P \leq 1/c$

$P_{\text{fail}} \leftrightarrow$ Area Fungibility

- What tradeoffs are possible?



Example: $\text{erfc}(A^{1/4})$



Only for some functions

Function	Effective Region
$p(A) = \frac{1}{A^\gamma}$	$\gamma \leq 5$
$p(A) = \exp(-A^\gamma)$	$\gamma \leq 0.48$
$p(A) = \text{erfc}(A^\gamma)$	$\gamma \leq 0.27$

Conclusions

Reliability \leftrightarrow Space, Time, Energy
Fault Tolerance can save resources

- CMOS?

$$P_{fail} = \text{erfc} \left(1 / \sqrt{\frac{K'_w(\gamma)}{v_{th}^2} \frac{1}{TP\gamma} + \frac{K'_f}{v_{th}^2} \frac{1}{TA}} \right)$$

- Neurons