The Quantum Schur Transform

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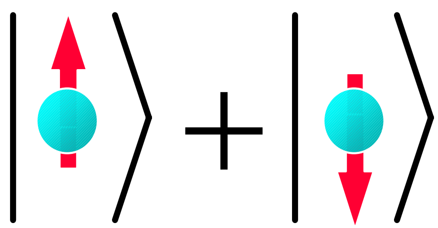
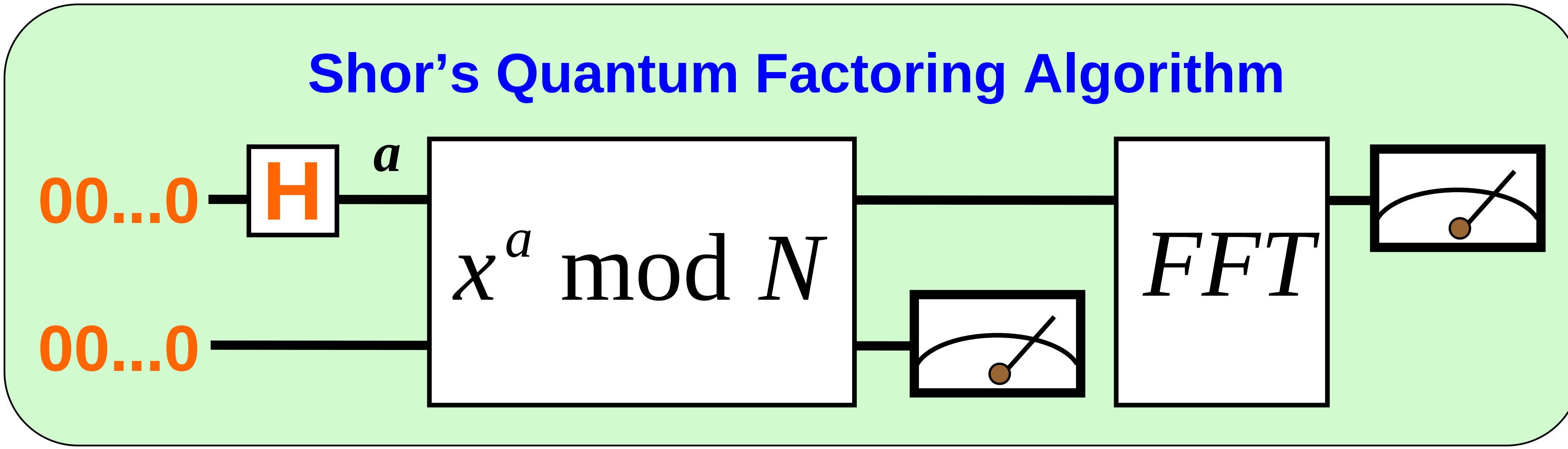
Aram Harrow
Univ. of Bristol, CS



I. Chuang
MIT, Physics

The Search for new Q. Algorithms



- **Current algorithms:**

- Deutsch-Jozsa '92: f(0)$\oplus$ f(1)
- Simon '94: period finding
- Shor '95: factoring
- Kitaev '96: hidden subgroup
- Grover '96: search
- Hallgren '02,...: Pell's equation $ax^2+1=y^2$

QFT Based

Beyond Shor's Algorithm?

- ⌘Exponential speedup algorithms
  - Is there any structure beyond the Quantum Fourier Transform (abelian Hidden subgroup methods)?
- ⌘QFT: Extract global properties
  - New result: the quantum Schur transform

(Bacon, Chuang, Harrow, Phys. Rev. Lett., to appear Oct. 2006)

Symmetries of $(\mathbb{C}^d)^{\otimes n}$

- Problem: What are the global properties of n copies of $|\psi\rangle$?
- Example: Two spins under $U \otimes U$ – singlet or triplet

λ "total spin"

"spin projection" q

$$|0,\ 0\rangle_{\mathrm{Sch}} = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

antisymmetric

$$|1,\ 1\rangle_{\mathrm{Sch}} = |11\rangle$$

$$|1,\ 0\rangle_{\mathrm{Sch}} = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|1, -1\rangle_{\mathrm{Sch}} = |00\rangle$$

symmetric

- Fact: λ and q are insufficient for n>2; also need perm. p

The Schur Transform

- Fact: $(\mathbb{C}^d)^{\otimes n} \cong \bigoplus_\lambda \mathcal{Q}_\lambda \otimes \mathcal{P}_\lambda$

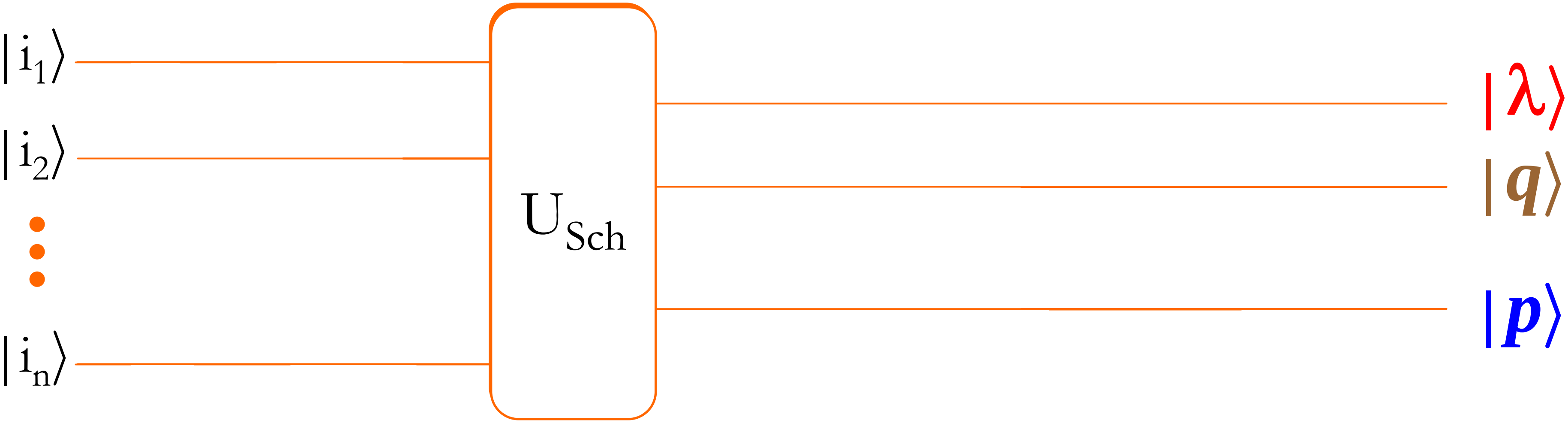
Schur Duality: Global properties of $|\psi\rangle^{\otimes n}$ under $U^{\otimes n}$ and S_n are described by λ, q, p

- Challenge: How can one transform

$$|i_1 i_2 \ldots i_n\rangle \to \Sigma_\lambda \; |\lambda, q_\lambda, p_\lambda\rangle \;?$$

Schur transform results

- Status: Efficient q. circuit for Schur transform constructed
  (Bacon, Chuang, & Harrow, quant-ph/0407082; PRL Oct'06)
- Input: n copies of $|\psi\rangle$
- Output: Total "spin" and symmetry irrep. classification



Schur transform: Applications

Universal entanglement concentration:

Given $|\psi_{AB}\rangle^{\otimes n}$, Alice and Bob both perform the Schur transform, measure λ, discard $\mathcal{Q}_\lambda$ and are left with a maximally entangled state in $\mathcal{P}_\lambda$ equivalent to $\approx nE(\psi)$ EPR pairs.

Universal data compression:

Given $\rho^{\otimes n}$, perform the Schur transform, weakly measure λ and the resulting state has dimension $\approx \exp(nS(\rho))$.

State estimation:

Given $\rho^{\otimes n}$, estimate the spectrum of ρ, or estimate ρ, or test to see whether the state is $\sigma^{\otimes n}$.