Fundamental limits to sensing and control

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Introduction: Quantum sensing and measurement

• At bottom, all systems are quantum mechanical. Quantum mechanics sets limits to the accuracy of measurement.

Applications

• Measurement of position (radar, LIGO, GPS)

• Measurement of time (quantum clocks, time of arrival measurement, GPS)

• Measurement of charge, electromagnetic field, etc.

Technologies of precision measurement are pushing up against their quantum limits:

What are those limits and how can we attain them?

Two tools:

1. Heisenberg uncertainty principle: $\Delta E \Delta t \geq \pi\hbar/2$.
2. Margolus-Levitin theorem: $E\Delta t \geq \pi\hbar/2$, where $E = \langle H \rangle - E_0$ is the average energy above the ground state energy E_0.

Example: A spin in a magnetic field. The ground state, $|\uparrow\rangle$ has energy 0 and the excited state $|\downarrow\rangle$ has energy $\hbar\omega = 2\mu B$, where μ is the magnetic moment of the spin and B is the strength of the field.

The state $|\rightarrow\rangle = (1/\sqrt{2})\big(|\uparrow\rangle + |\downarrow\rangle\big)$ has average energy $E = \hbar\omega_2$ and spread in energy $\Delta E = E$.

Over time Δt, this state evolves as $|\rightarrow\rangle \Longrightarrow |\uparrow\rangle + e^{-i\omega\Delta t}|\downarrow\rangle\big)$.

When $\Delta t = \pi/\omega = \pi\hbar/2E = \pi\hbar/2\Delta E$, the spin flips.

Atomic clocks

• Atomic clocks consist of collections of spins, coupled to an oscillator.

• If each spin has average energy E and spread in energy ΔE, then the average energy of n spins is nE and the spread in energy of n uncorrelated spins is $\sqrt{n}\Delta E$.

• The accuracy of a clock with n uncorrelated spins goes as $\Delta t_C = \pi/\omega\sqrt{n}$. This is the standard quantum limit for clock accuracy.

• The accuracy of a clock with n *entangled* spins goes as $\Delta t_Q = \pi/n\omega = \pi\hbar/2nE = \pi\hbar/2n\Delta E$.

$\Longrightarrow$ Entanglement beats the standard quantum limit!

• The global positioning system (GPS) consists of clocks in space that map out the geometry of space and time. *How accurately can GPS map out some volume of spacetime?*

• Consider a spacetime volume consisting of a sphere of radius r followed over time t. Suppose the clocks and signals within this volume have energy E. Then the total number of ticks of clocks and clicks of detectors that can take place within this volume is $\# \leq 2Et/\pi\hbar$.

• To function as part of the GPS network, the volume cannot be a black hole. So $r \geq R_S = 2GE/c^4$ (since $E = mc^2$).

• As a result

$$\# \leq \frac{rtc^4}{\pi\hbar G} = \frac{rt}{\pi \ell_P t_P}$$

where $\ell_P = \sqrt{\hbar G/c^3}$ is the Planck length and $t_P = \sqrt{\hbar G/c^5}$ is the Planck time: *The Quantum Geometric Limit.*

Many open questions remain:

What are the ultimate limits to sensing and measurement in the presence of noise? (Here there are examples of situations in which quantum techniques do more than $\sqrt{n}$ better than semiclassical techniques.)

How do we attain those limits?

Problems of precision and speed of quantum control obey similar limits in the semiclassical setting: what are the ultimate limits of control?

• The resolutions of these questions are closely related to similar questions in quantum communication theory, such as the capacity of the noisy bosonic channel and the additivity of quantum channel capacity.