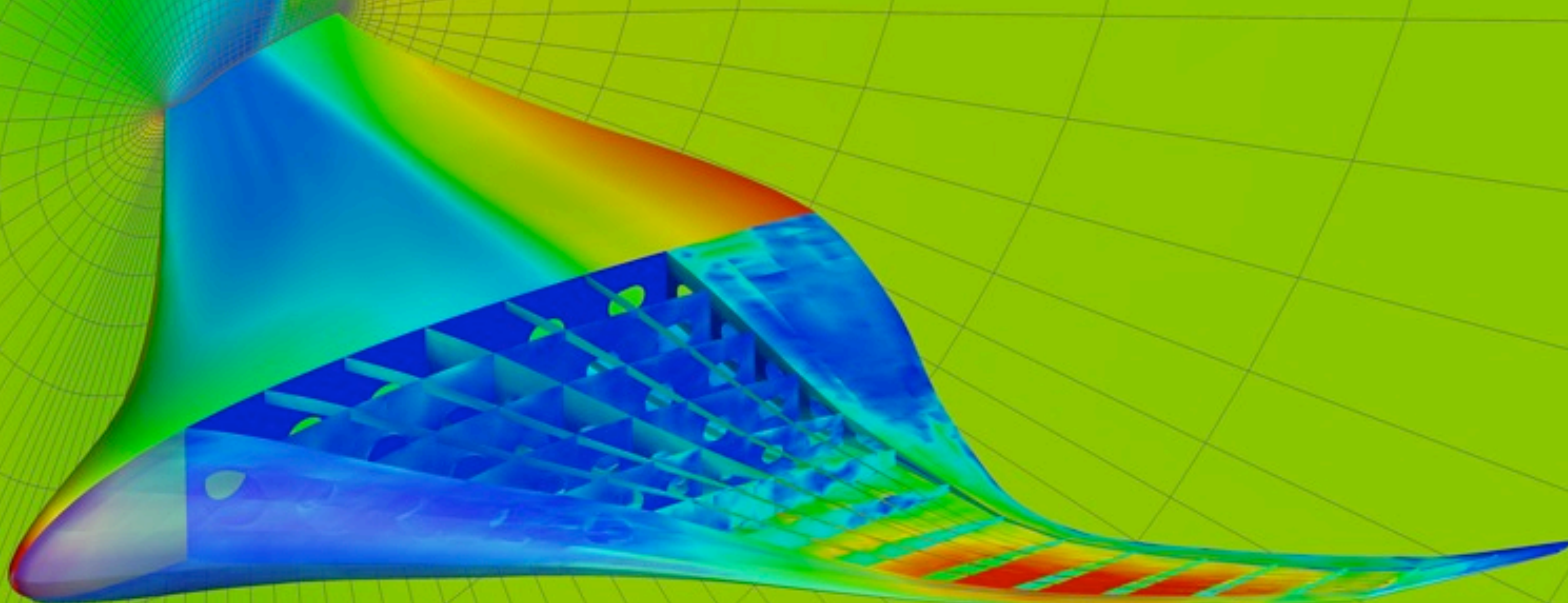
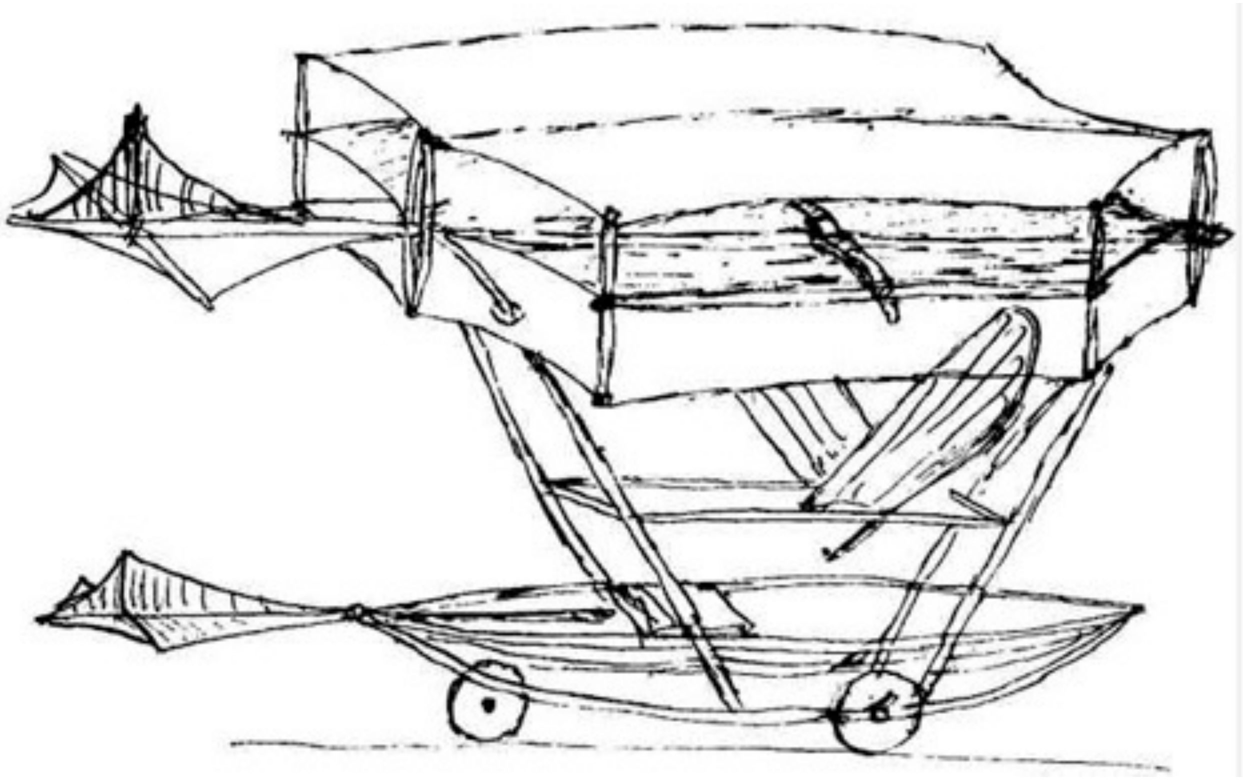
Multidisciplinary Design Optimization



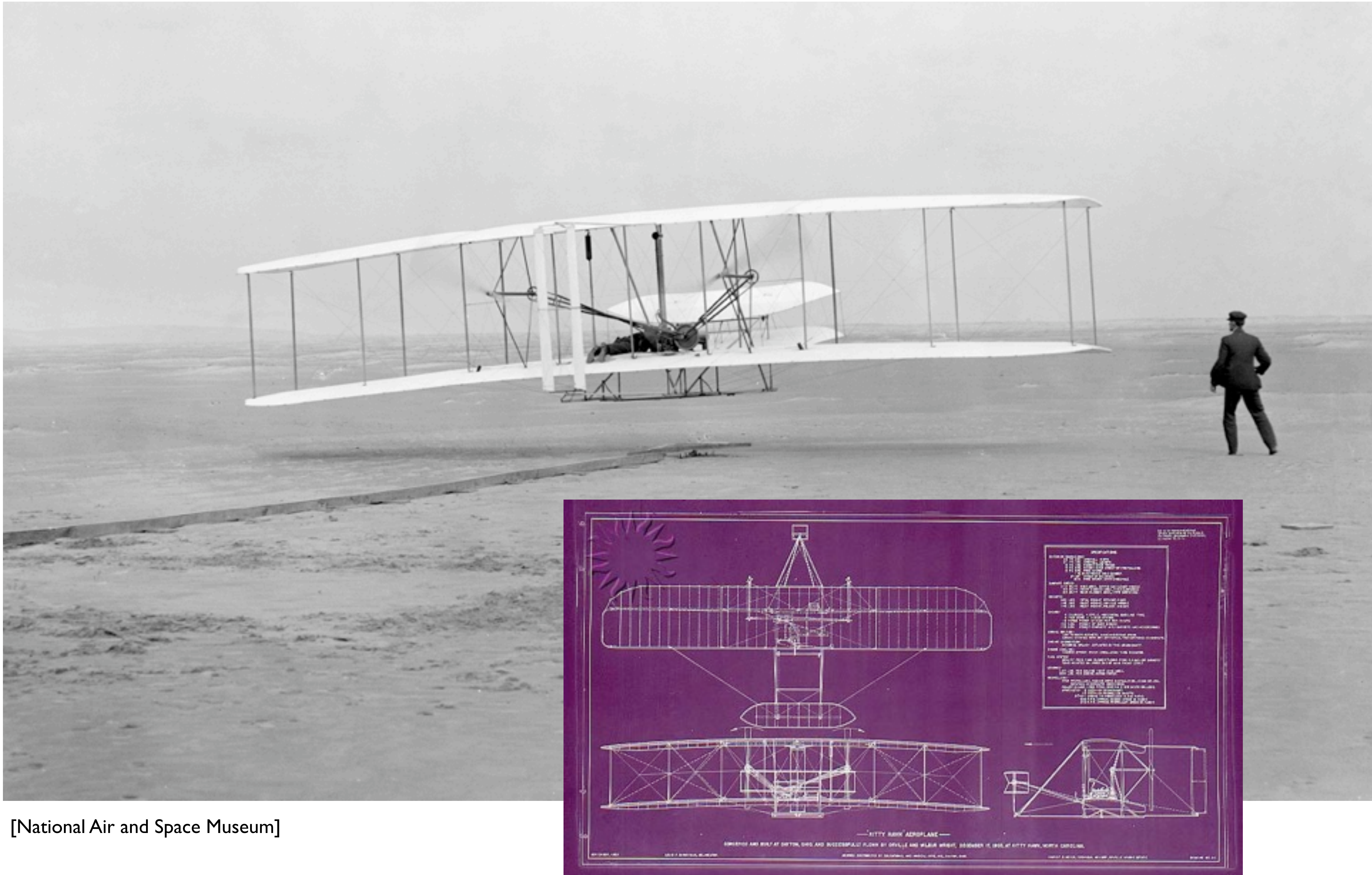
Joaquim R. R. A. Martins
Multidisciplinary Design Optimization Laboratory
http://mdolab.engin.umich.edu/
University of Michigan

7th International Fab Lab Forum and Symposium on Digital Fabrication
Lima, Peru, August 18, 2011
(Remote presentation)

Sir George Cayley

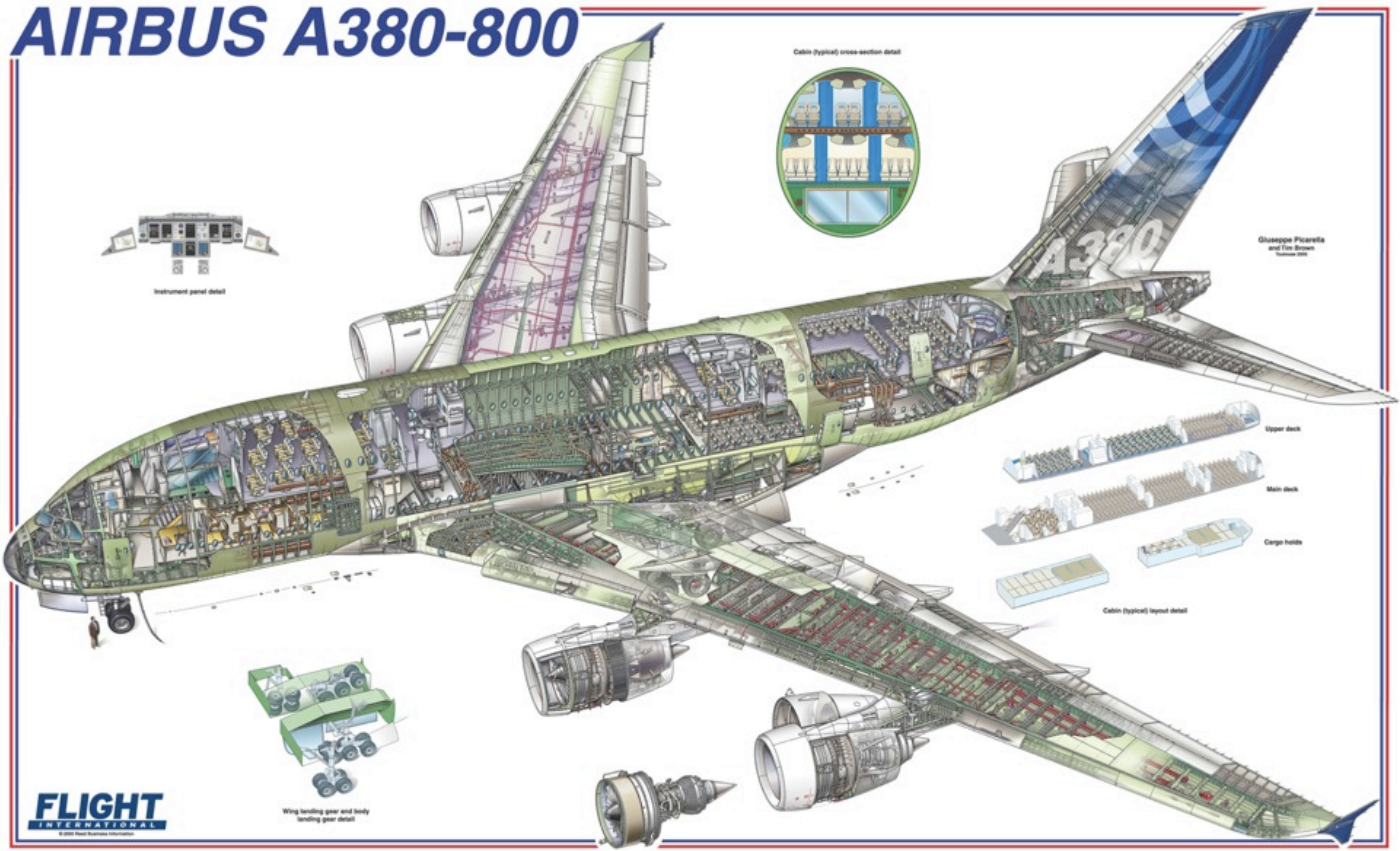


The Dawn of Multidisciplinary Design



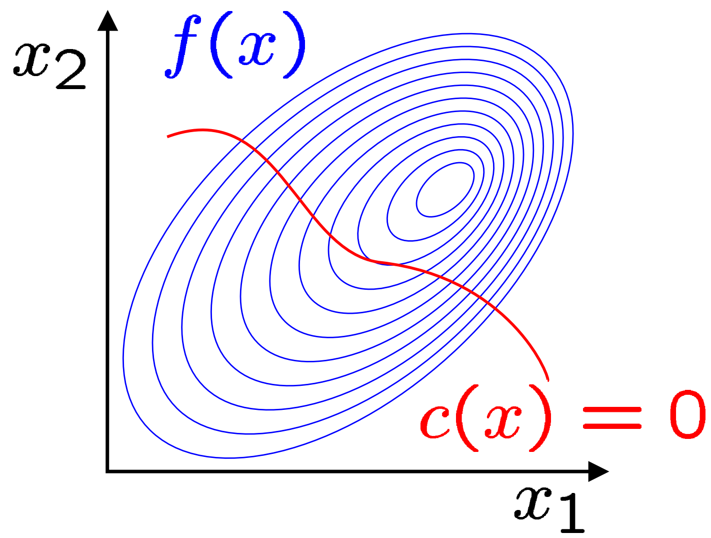
[National Air and Space Museum]

Current Multidisciplinary Design



[Flight International]

What is Optimization?



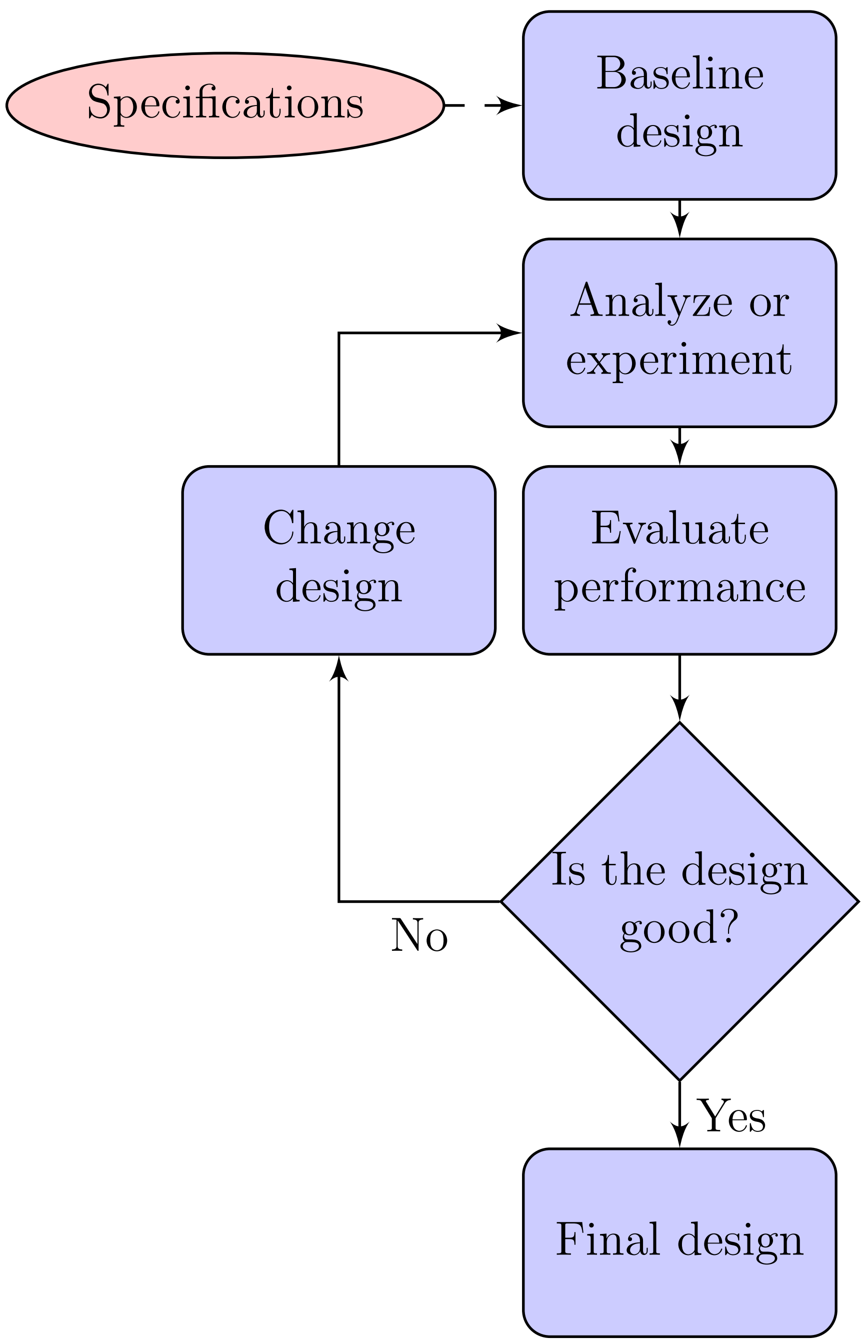
$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{by varying} & x \in \mathbb{R}^n \\ \text{subject to} & c_j(x) \geq 0, \quad j = 1, 2, \ldots, m \end{array}$$

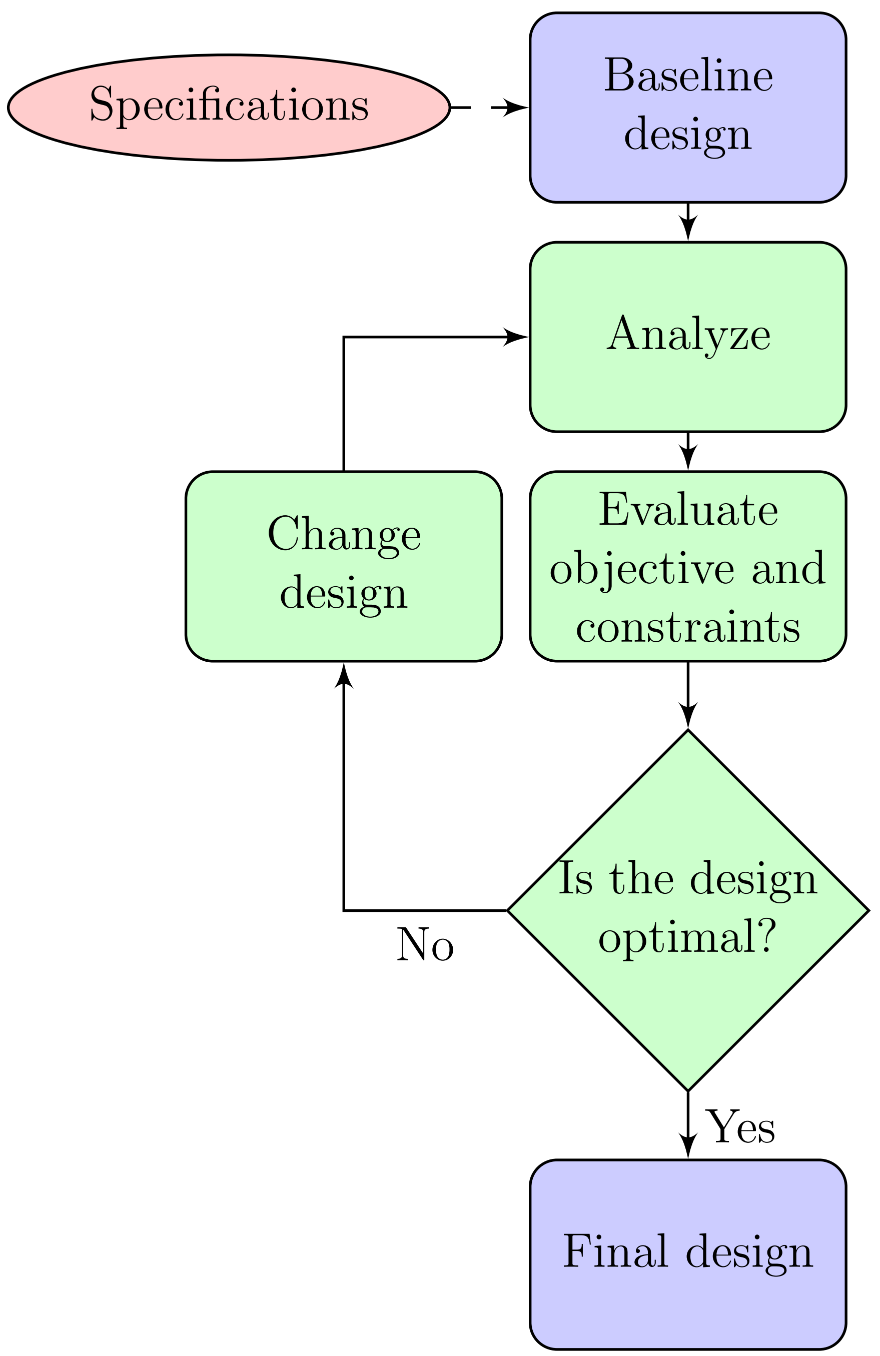
f : objective function, output (e.g. structural weight).

x : vector of design variables, inputs (e.g. aerodynamic shape); bounds can be set on these variables.

c : vector of inequality constraints (e.g. structural stresses), may also be nonlinear and implicit.

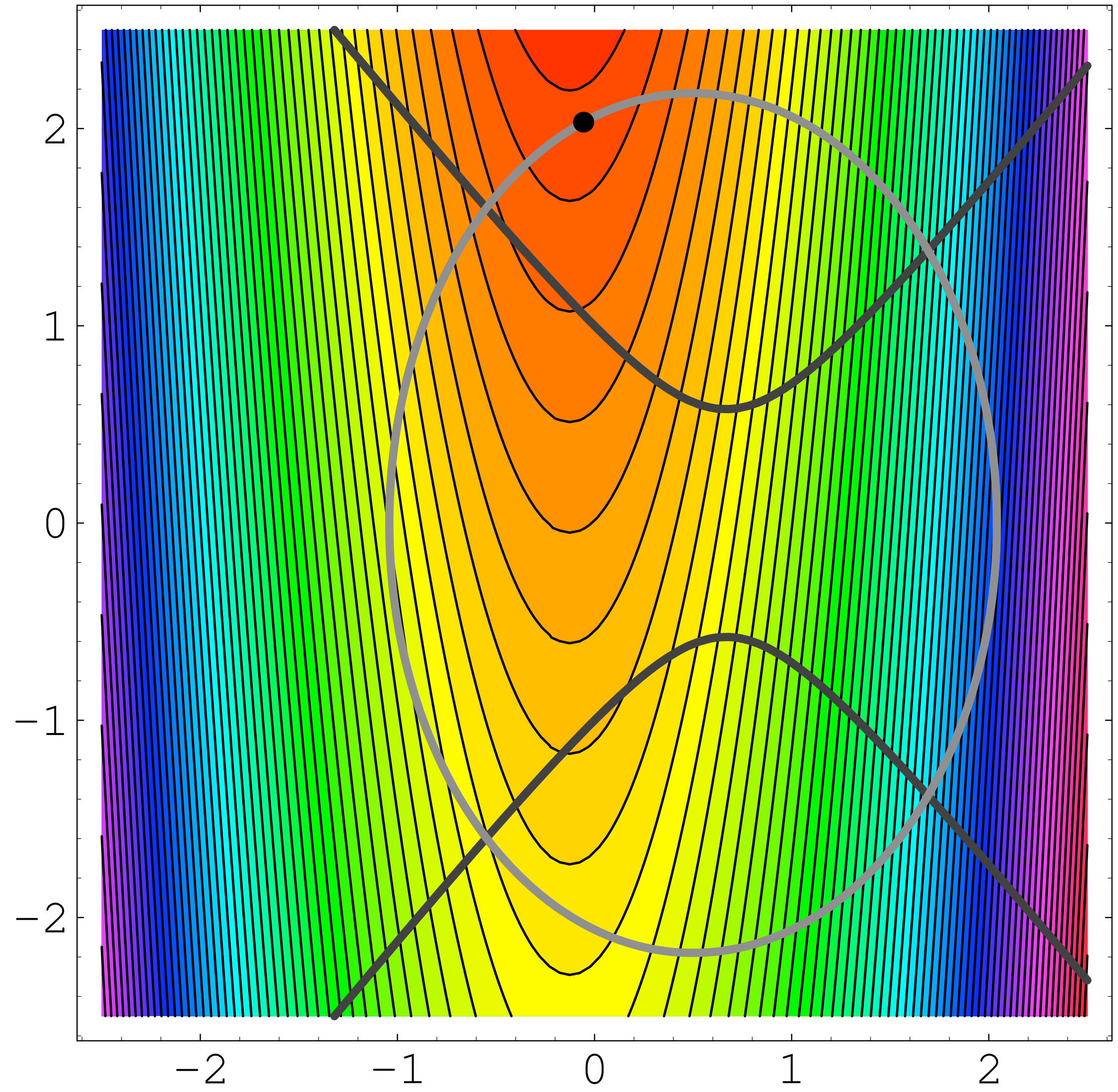
Conventional vs. Optimal Design Process





Numerical Optimization

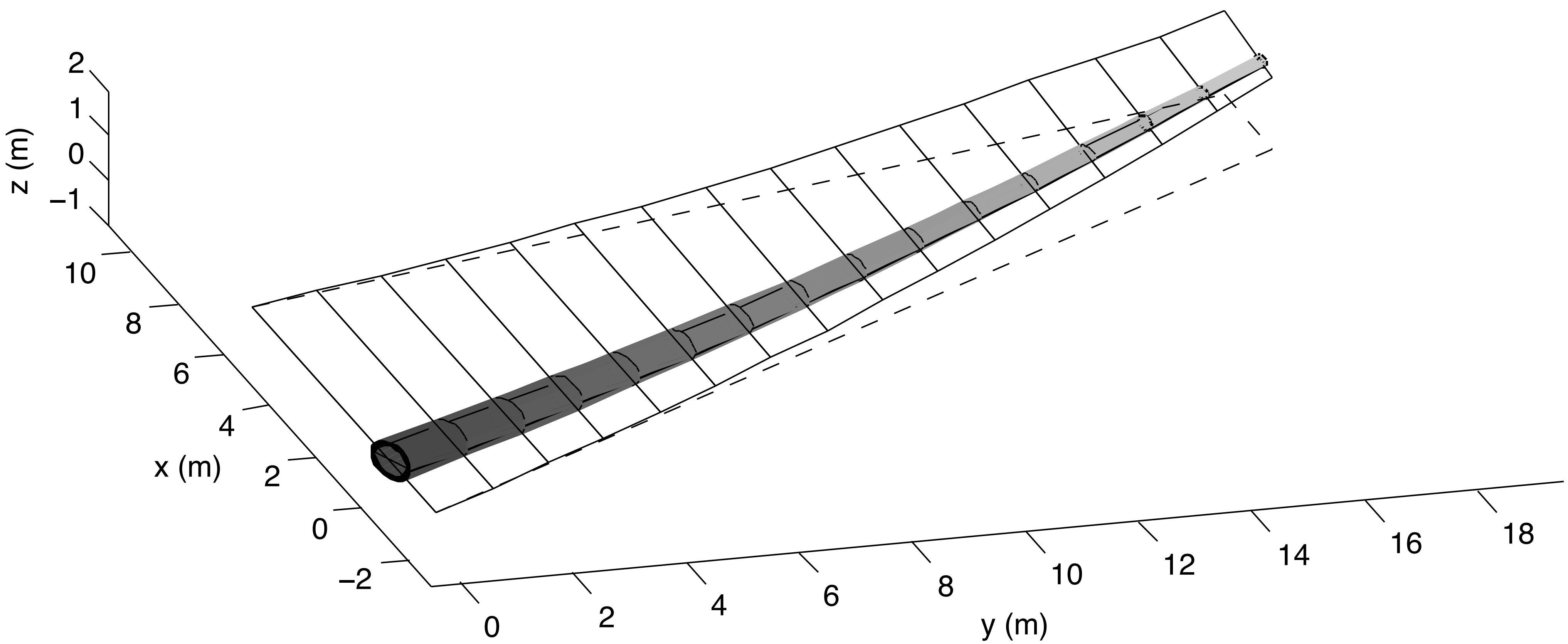
minimize	$f(x) = 4x_1^2 - x_1 - x_2 - 2.5$
by varying	x_1, x_2
subject to	$c_1(x) = x_2^2 - 1.5x_1^2 + 2x_1 - 1 \geq 0,$
	$c_2(x) = x_2^2 + 2x_1^2 - 2x_1 - 4.25 \leq 0$



MDO: A Wing Design Example

Maximize:

$$\text{Range} \propto \frac{L}{D} \ln\left(\frac{W_i}{W_f}\right)$$



Aerodynamics: Panel code computes induced drag. Variables: wing twist and angle of attack

Structures: Beam finite-element model of the spar that computes the displacements and stresses. Variables: element thicknesses

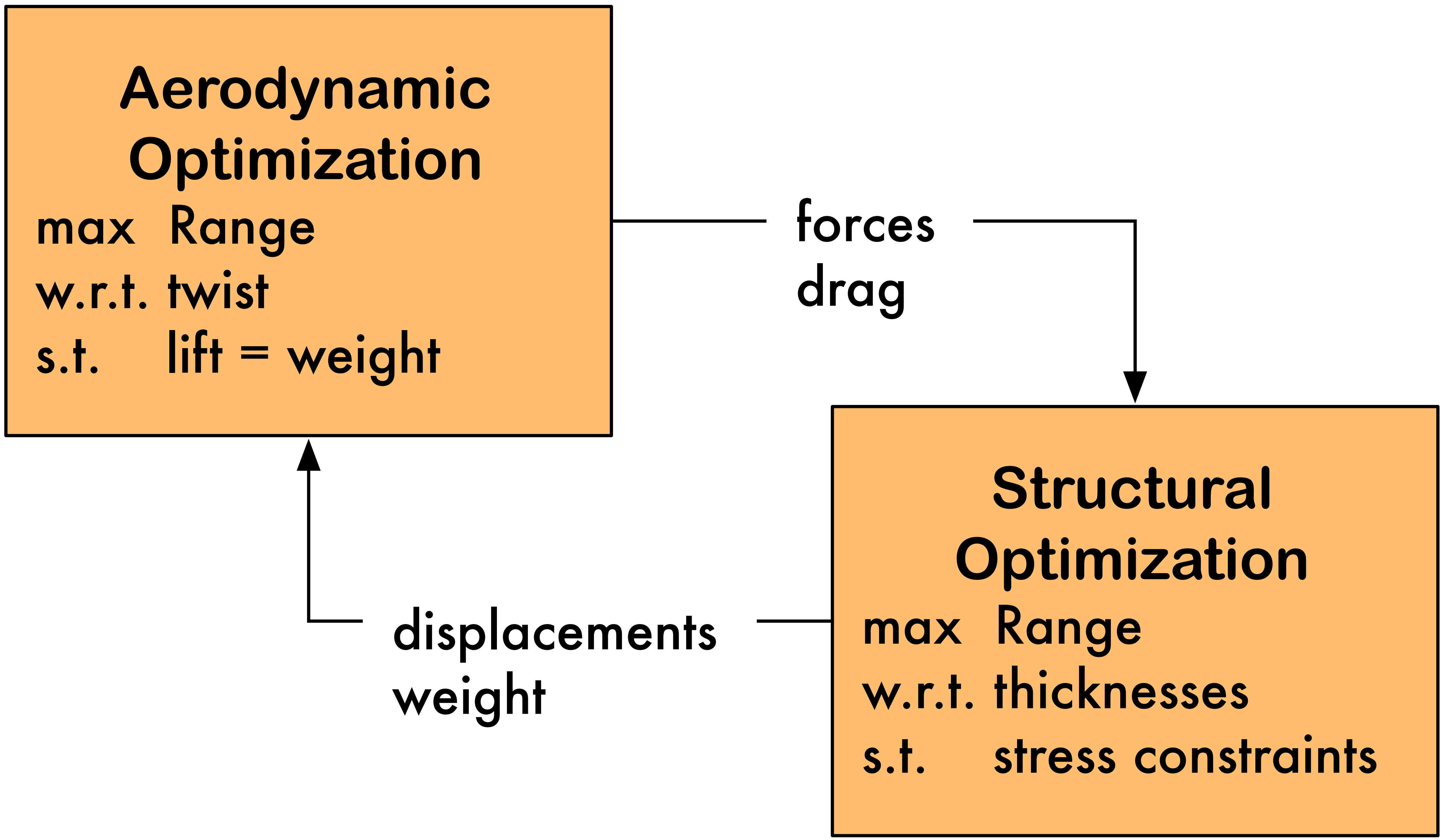
Aerostructural Coupling — Boeing 787

[airliners.net]

Aerostructural Coupling — Boeing 787

[airliners.net]

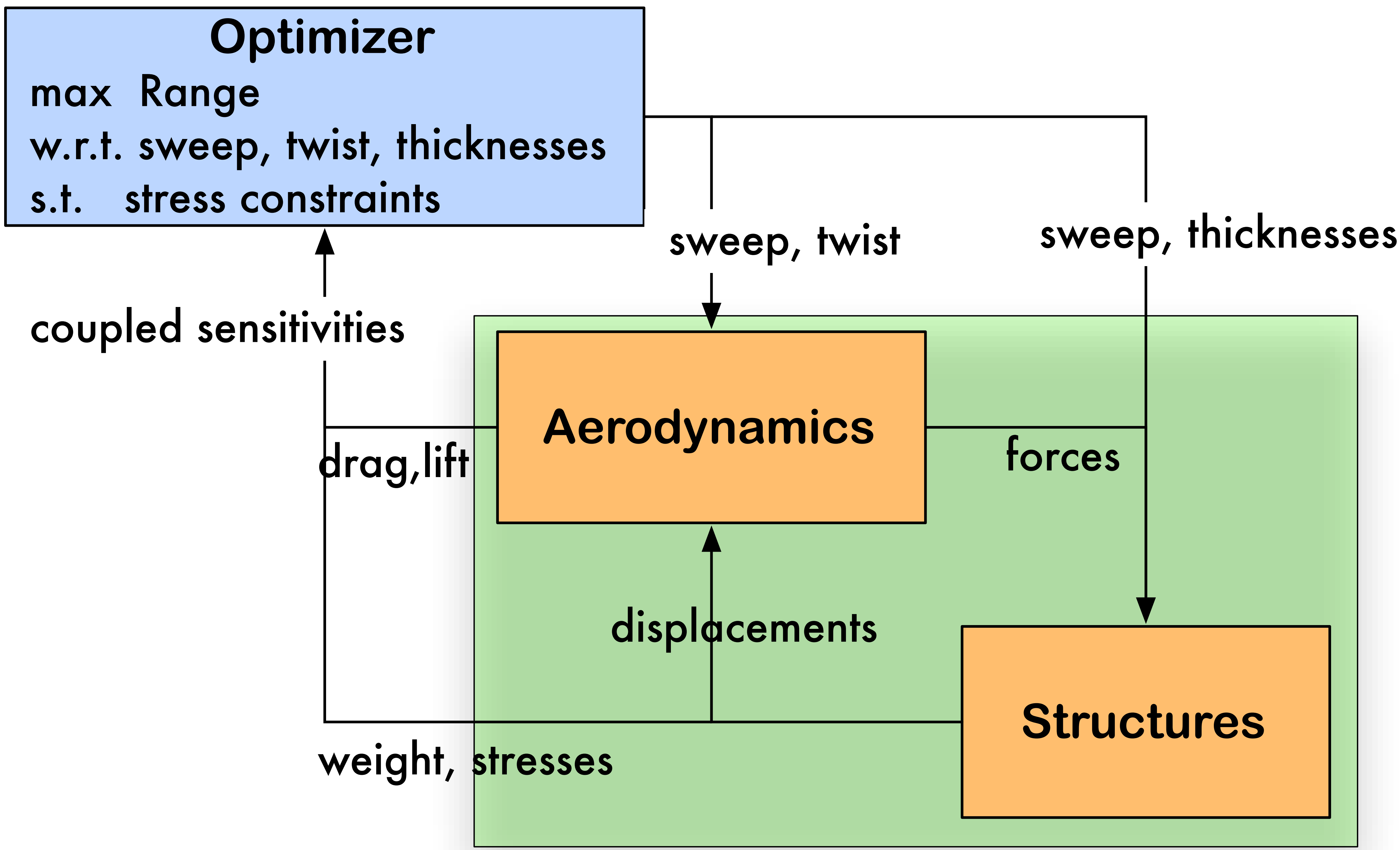
Sequential Optimization



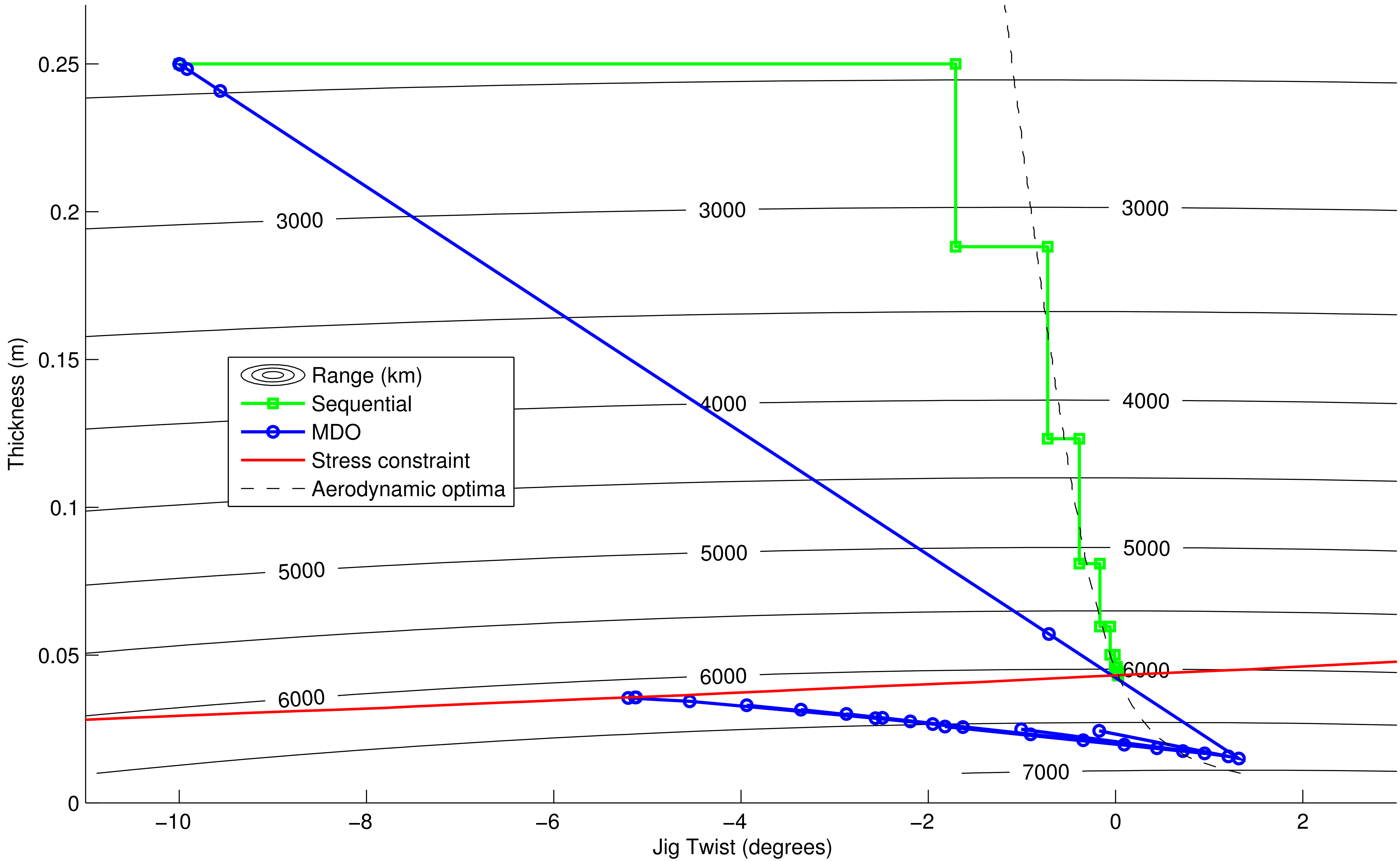
The final result is always an elliptic lift distribution

A Sound MDO Approach

The multidisciplinary feasible (MDF) method

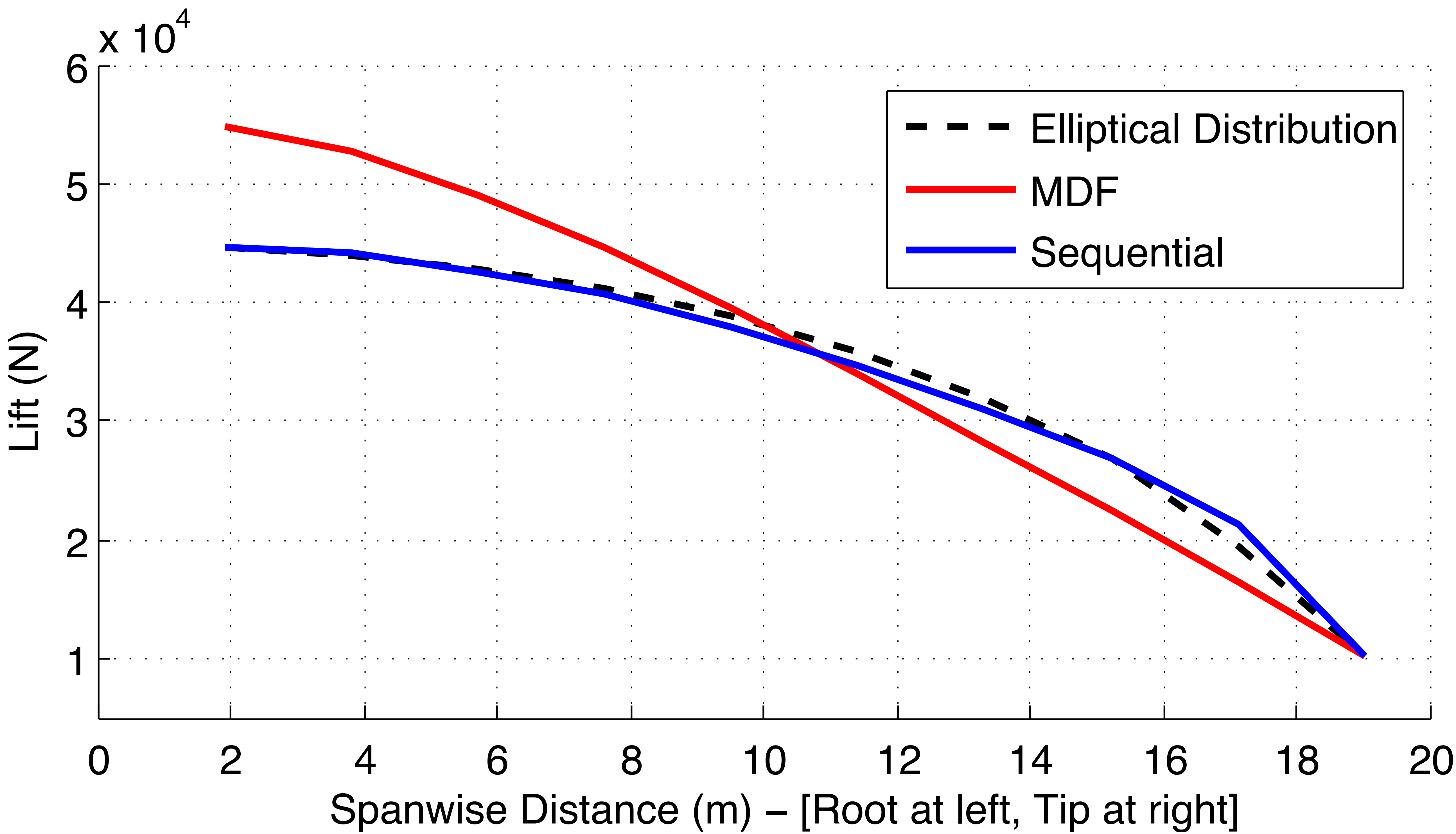


Sequential Optimization vs. MDO



[Chittick and Martins, *Structural and Multidisciplinary Optimization*, 2008]

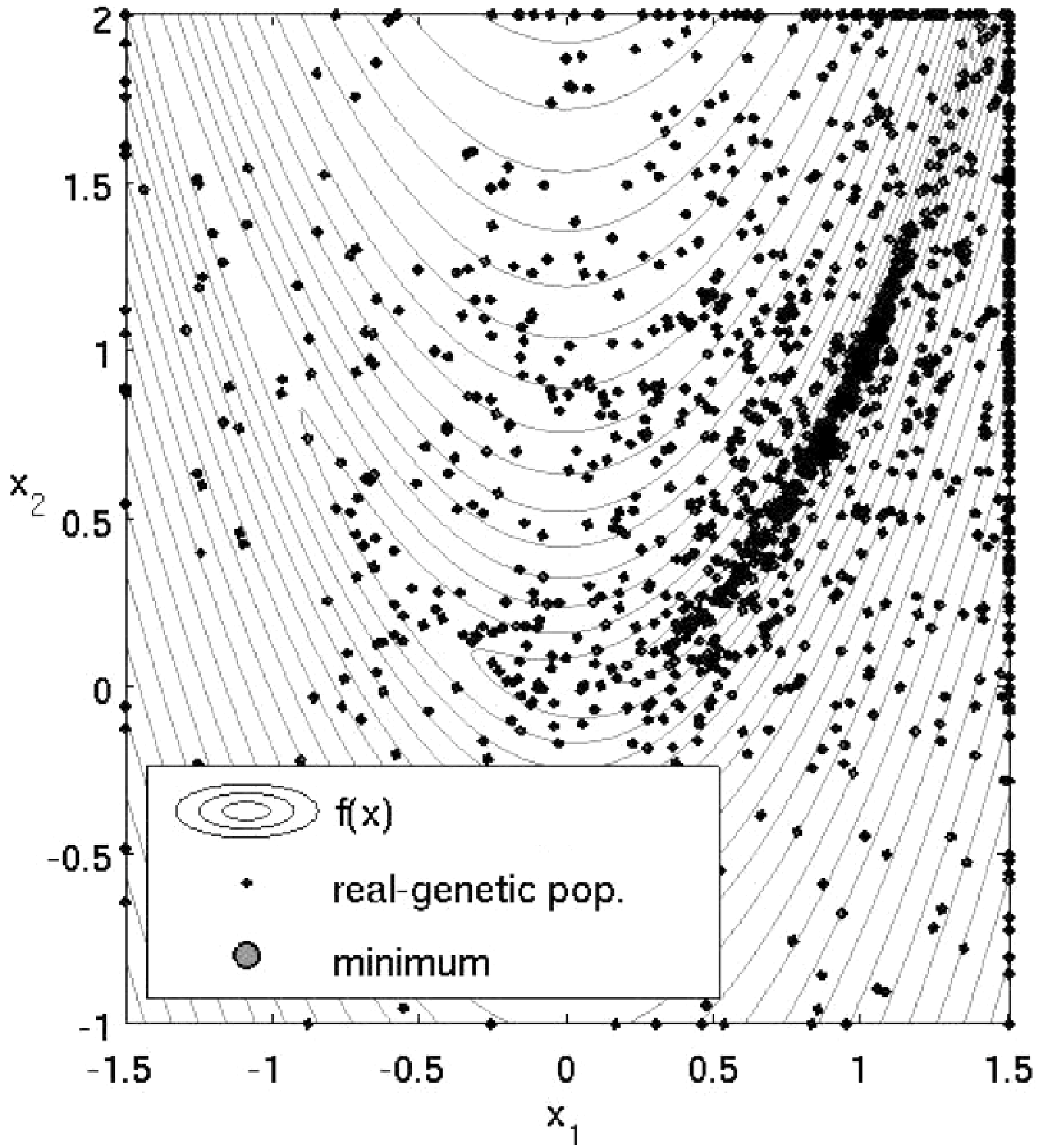
Sequential Optimization vs. MDF

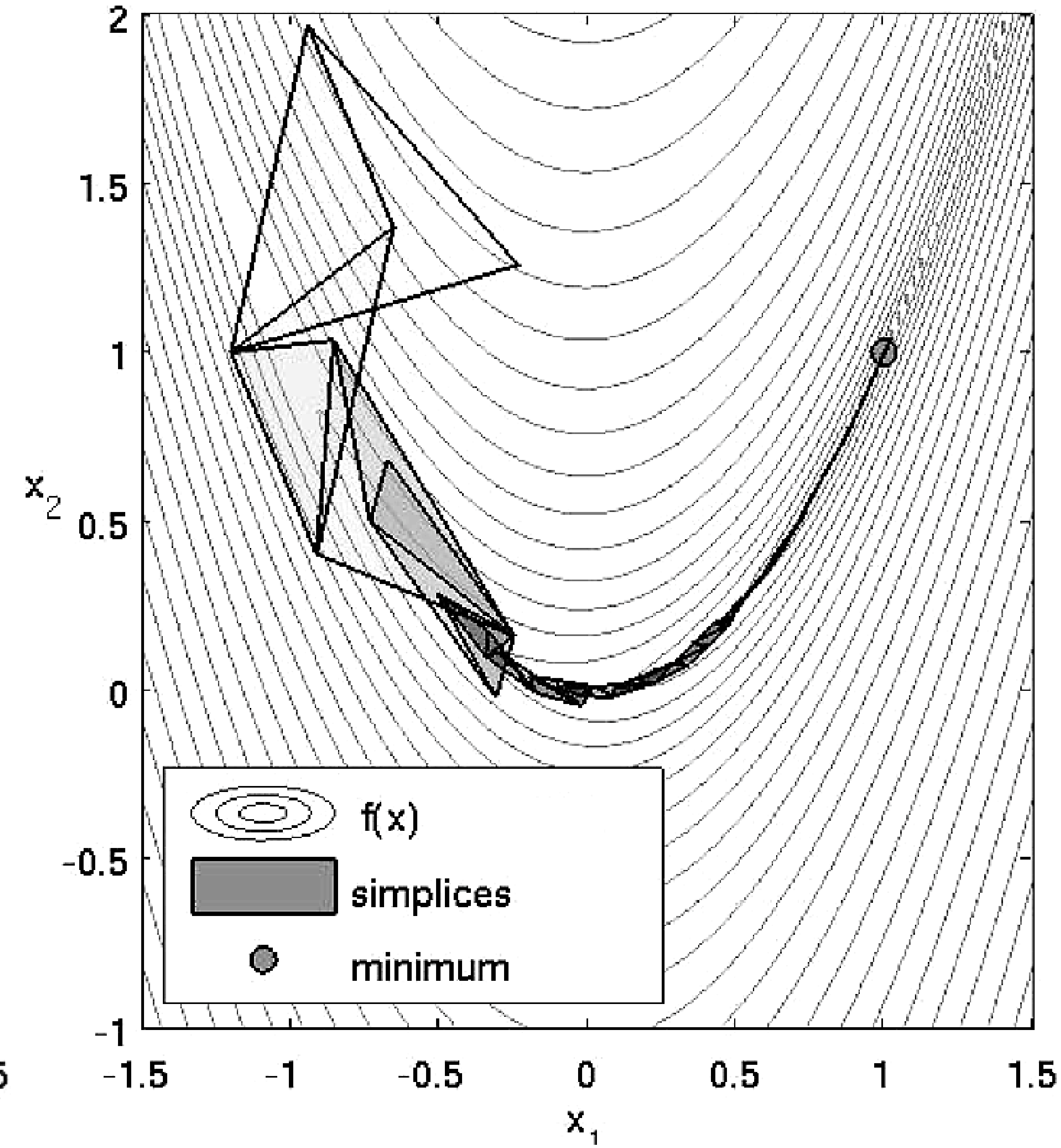


Optimization Methods

Engineering intuition

Optimization Methods: Gradient-Free

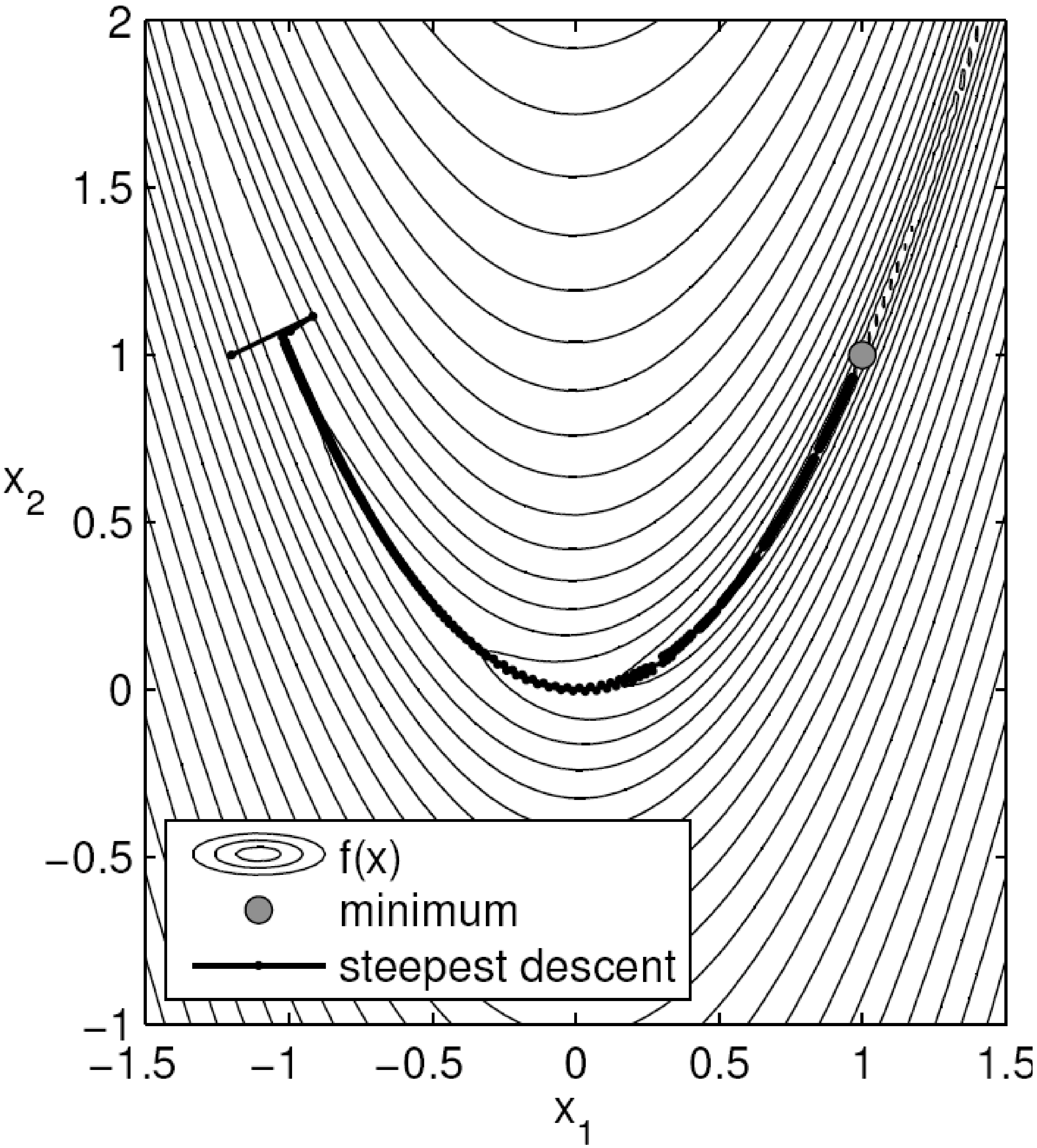




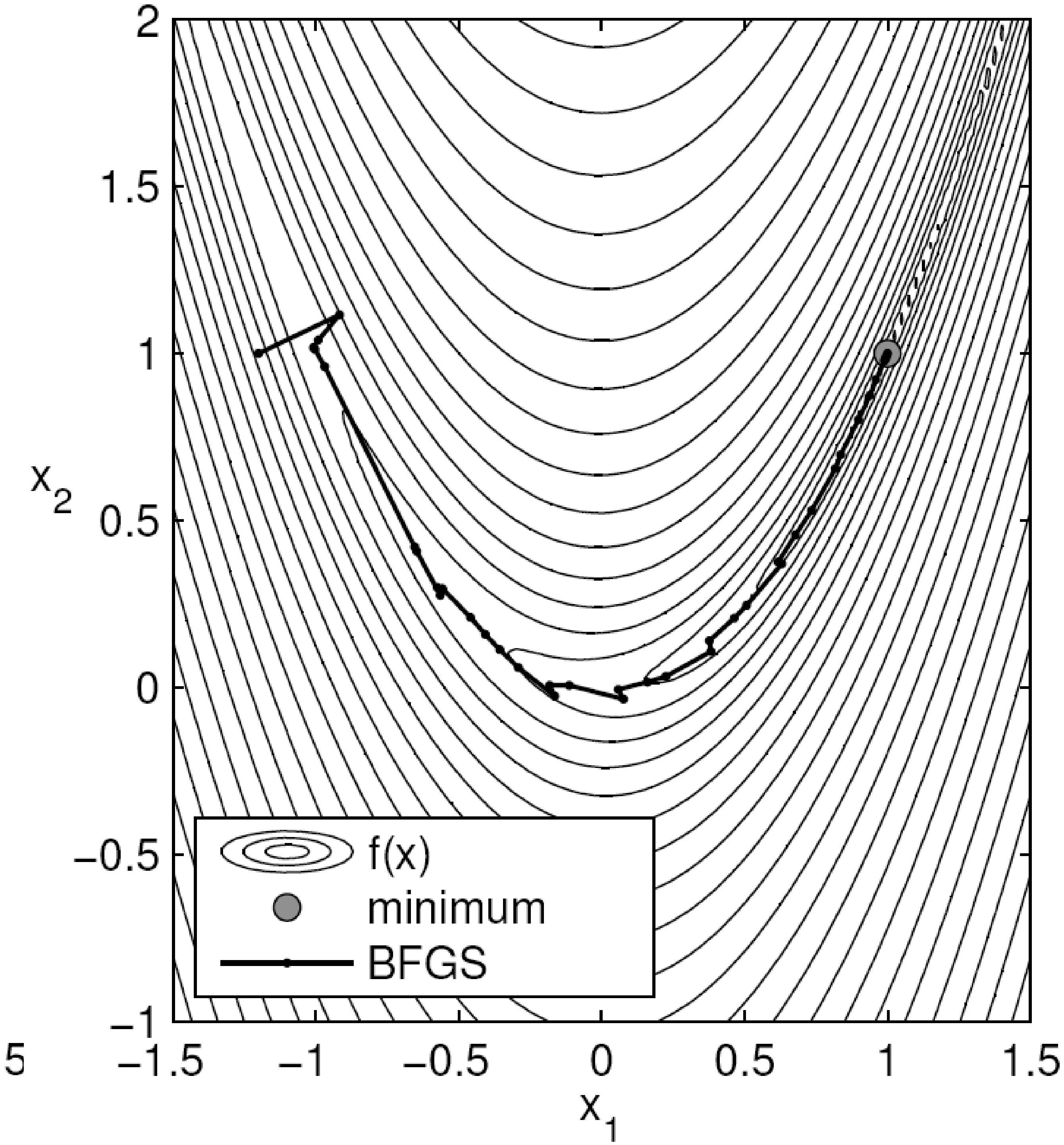
Genetic algorithms

Nelder-Mead simplex

Optimization Methods: Gradient-Based

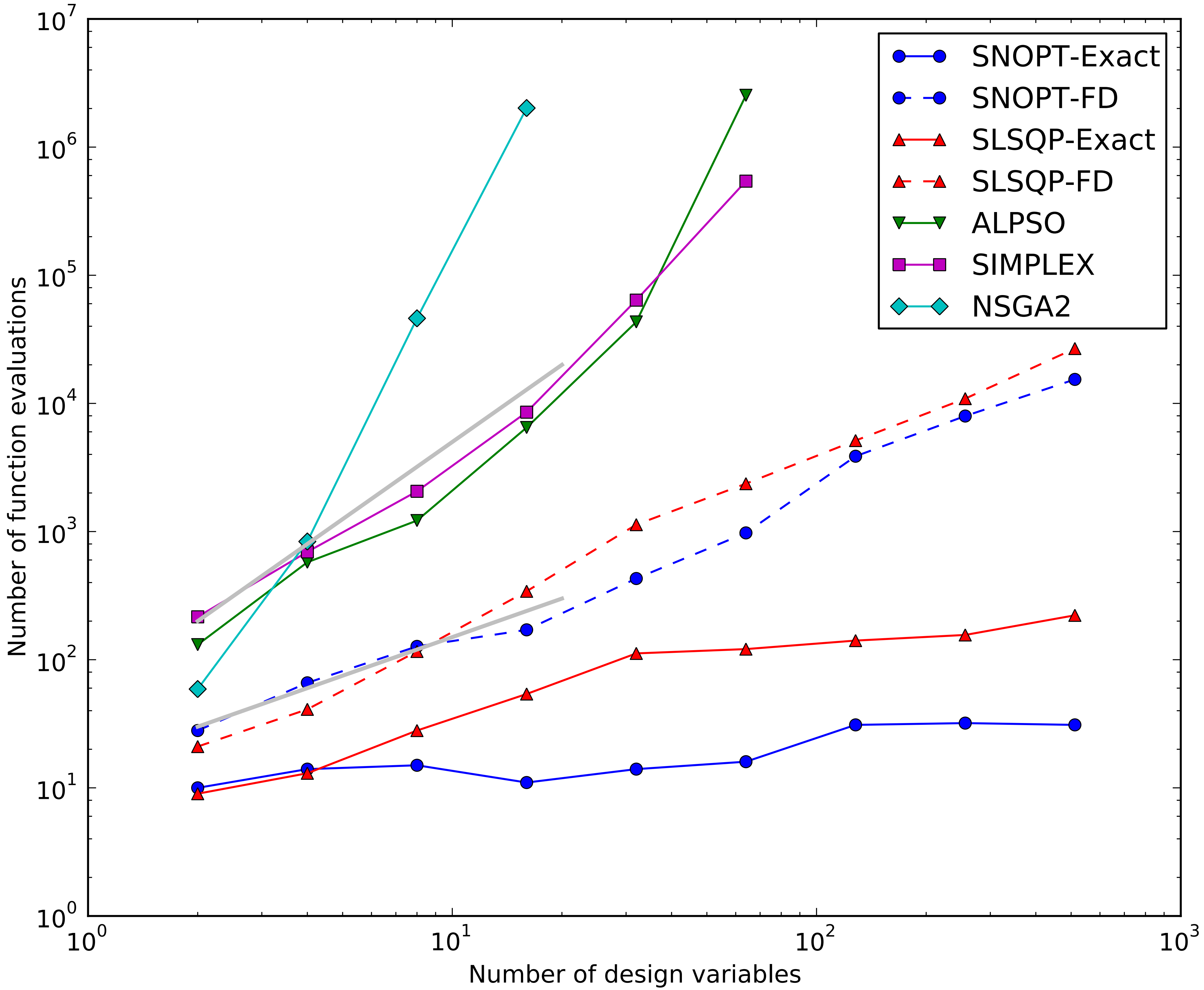


Steepest descent (1st order)

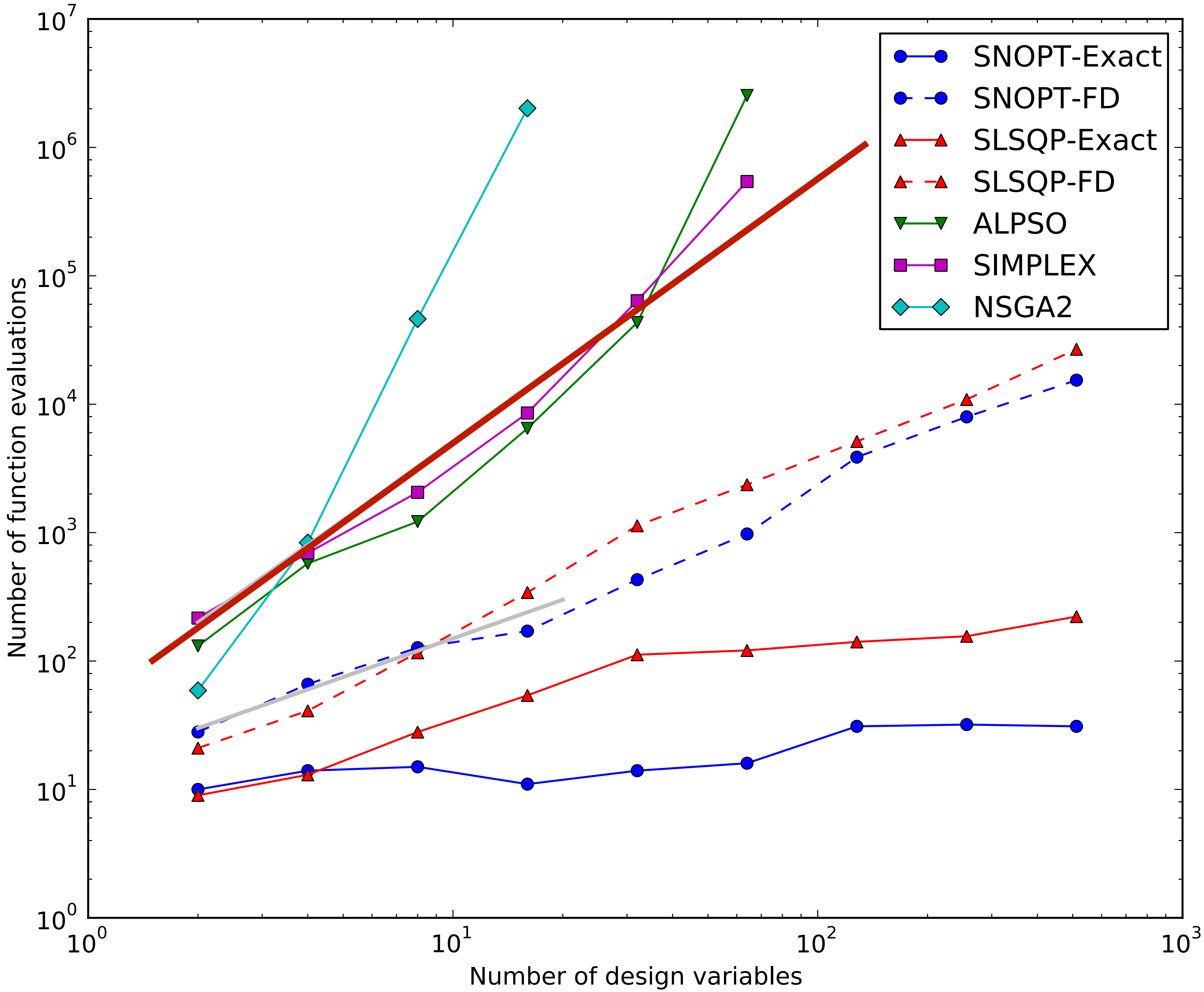


BFGS (2nd order)

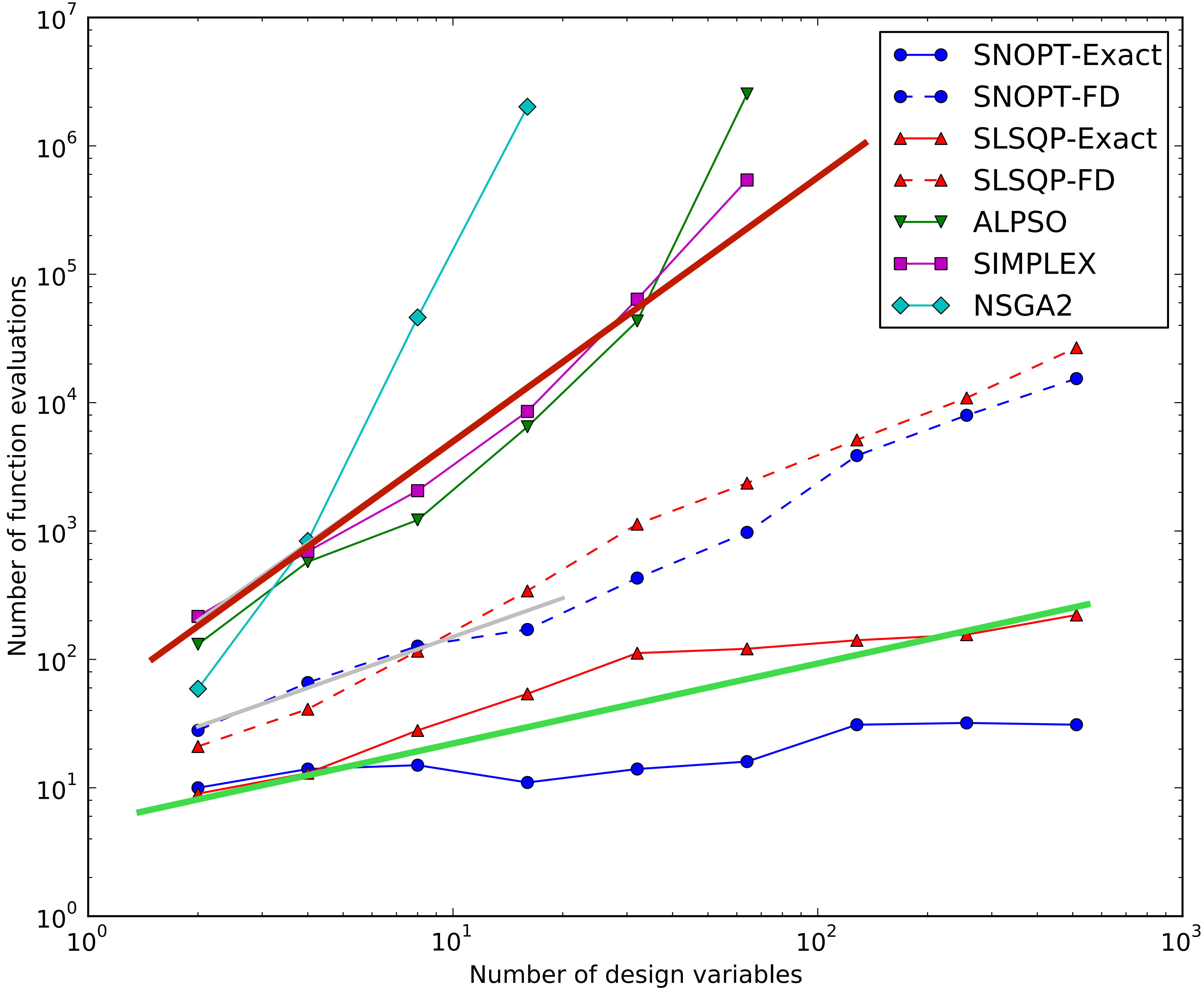
Optimization: Gradient-Based vs. Not



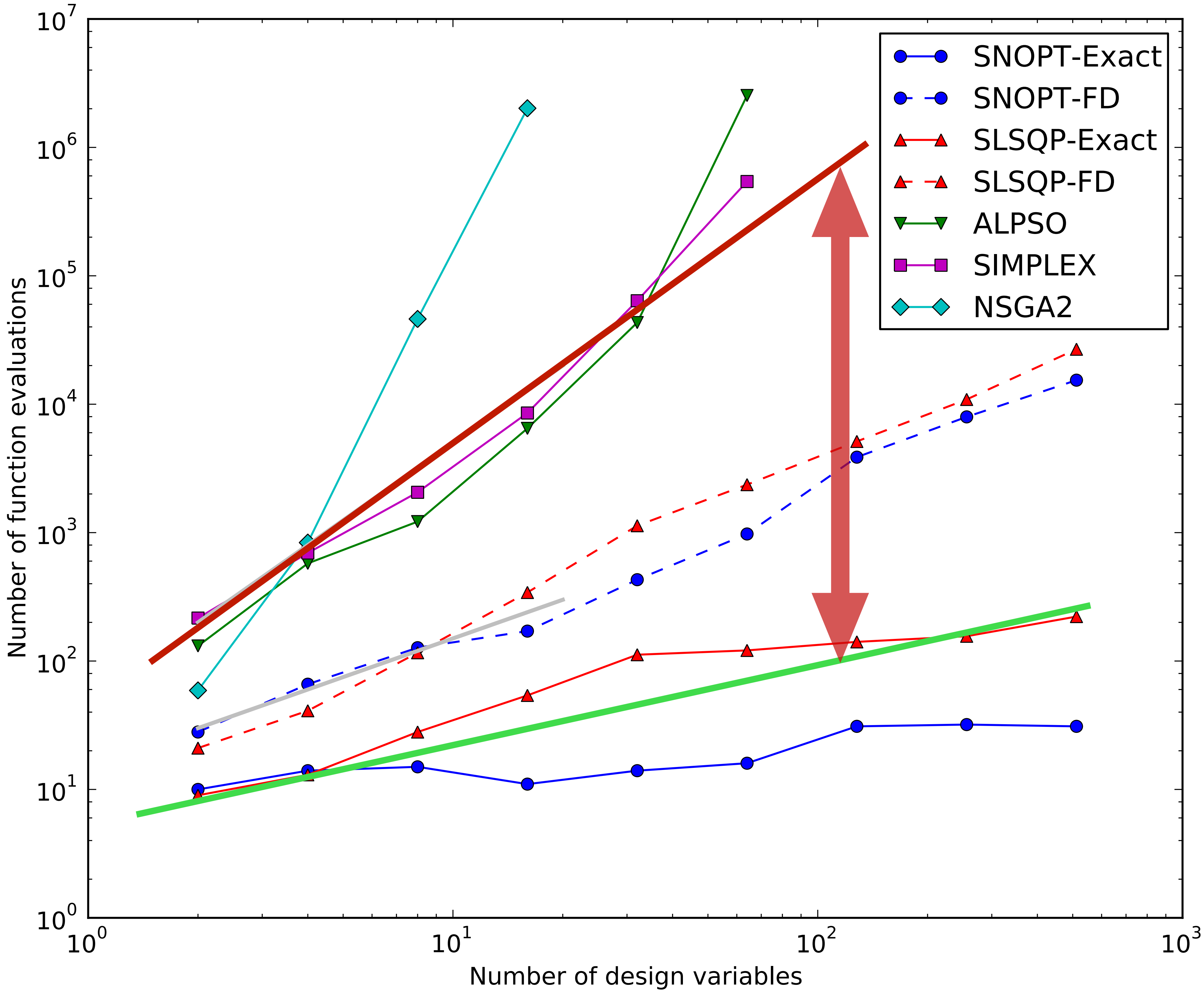
Optimization: Gradient-Based vs. Not



Optimization: Gradient-Based vs. Not

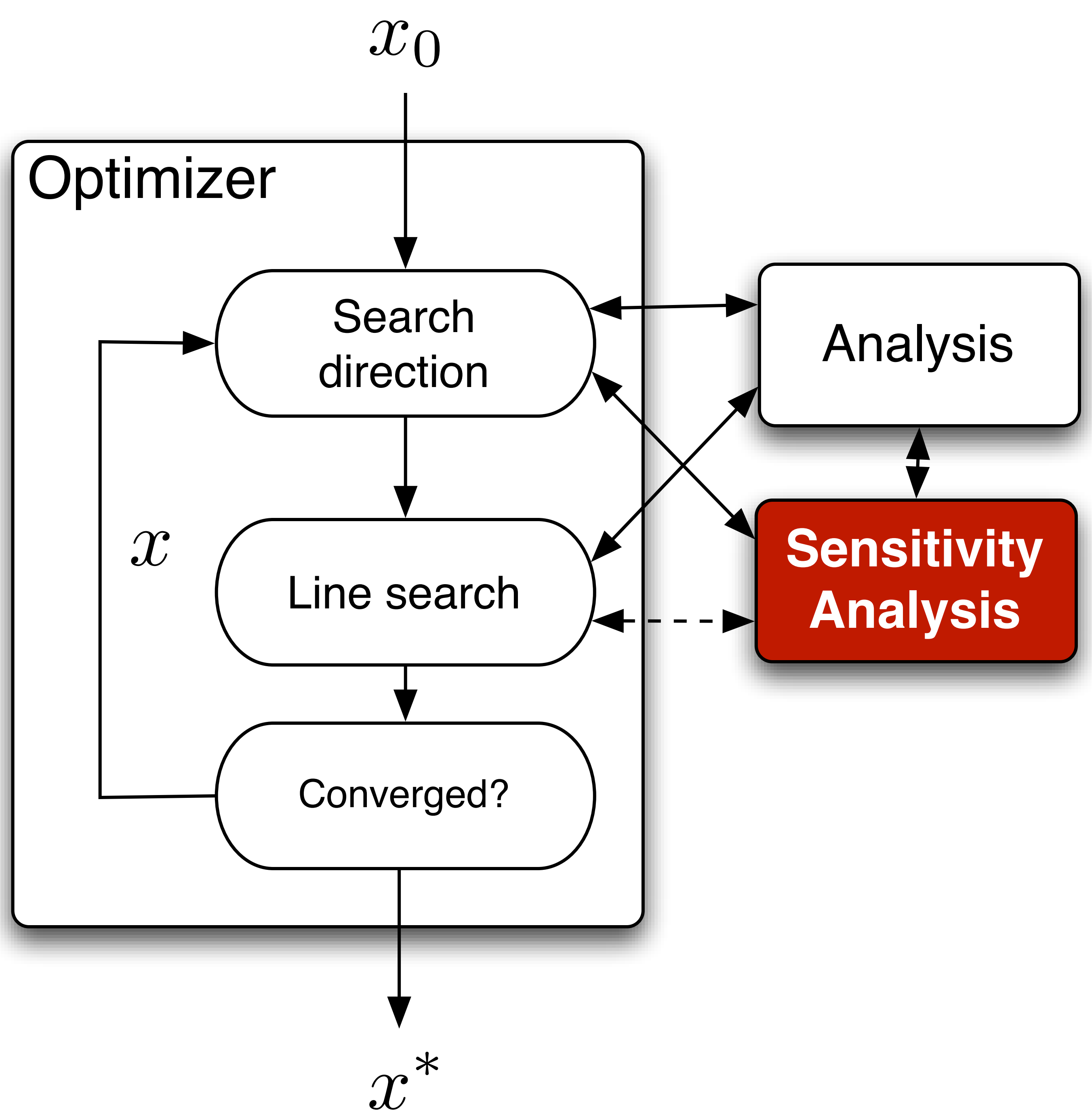


Optimization: Gradient-Based vs. Not



The Case for Efficient Sensitivity Analysis

- By default, most gradient-based optimizers use finite differences
- When using finite differences with large numbers of design variables, sensitivity analysis is the bottleneck
- Accurate sensitivities needed for convergence



Sensitivity Analysis Methods

Finite differences: very popular, easy to implement, but can be very inaccurate; need to run analysis for each design variable

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Complex-step method: accurate, easy to implement and maintain; need to run analysis for each design variable

$$f'(x) \approx \frac{\mathrm{Im}\left[f(x+ih)\right]}{h}$$

[Martins, Alonso and Sturdza, *ACM TOMS*, 2003]

Automatic differentiation: automatic implementation, accurate; cost can be independent of the number of design variables

(Semi-)Analytic Methods: efficient and accurate, long development time; cost can be independent of the number of design variables

Sensitivity Analysis Methods

Finite differences: very popular, easy to implement, but can be very inaccurate; need to run analysis for each design variable

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Complex-step method: accurate, easy to implement and maintain; need to run analysis for each design variable

$$f'(x) \approx \frac{\mathrm{Im}\left[f(x+ih)\right]}{h}$$

[Martins, Alonso and Sturdza, *ACM TOMS*, 2003]

Automatic differentiation: automatic implementation, accurate; cost can be independent of the number of design variables

(Semi-)Analytic Methods: efficient and accurate, long development time; cost can be independent of the number of design variables

Complex-Step Derivative Approximation

Like finite differences, can be derived from a Taylor series expansion, but use a complex step instead of a a real one:

$$f(x+ih) = f(x) + ihf'(x) - h^2\frac{f''(x)}{2!} - ih^3\frac{f'''(x)}{3!} + \ldots$$

- No subtractive cancellation
- Numerically exact for small enough step

[Martins, Alonso and Sturdza, *ACM TOMS*, 2003]

Complex-Step Derivative Approximation

Like finite differences, can be derived from a Taylor series expansion, but use a complex step instead of a a real one:

$$f(x+ih) = f(x) + ihf'(x) - h^2\frac{f''(x)}{2!} - ih^3\frac{f'''(x)}{3!} + \ldots$$

$$\Rightarrow \quad f'(x) = \frac{\mathrm{Im}\left[f(x+ih)\right]}{h} + h^2\frac{f'''(x)}{3!} + \ldots$$

- No subtractive cancellation
- Numerically exact for small enough step

[Martins, Alonso and Sturdza, *ACM TOMS*, 2003]

Complex-Step Derivative Approximation

Like finite differences, can be derived from a Taylor series expansion, but use a complex step instead of a a real one:

$$f(x+ih) = f(x) + ihf'(x) - h^2\frac{f''(x)}{2!} - ih^3\frac{f'''(x)}{3!} + \ldots$$

$$\Rightarrow \quad f'(x) = \frac{\mathrm{Im}\left[f(x+ih)\right]}{h} + h^2\frac{f'''(x)}{3!} + \ldots$$

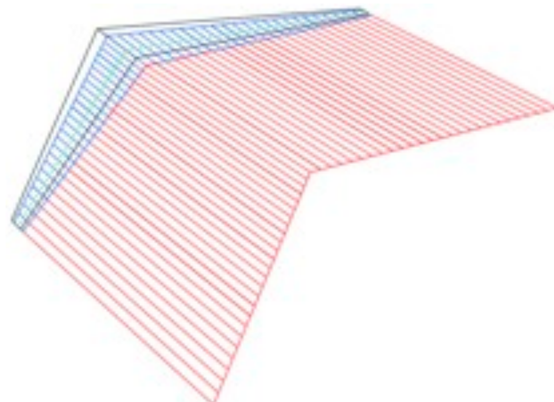
$$\Rightarrow \quad \boxed{f'(x) \approx \frac{\mathrm{Im}\left[f(x+ih)\right]}{h}}$$

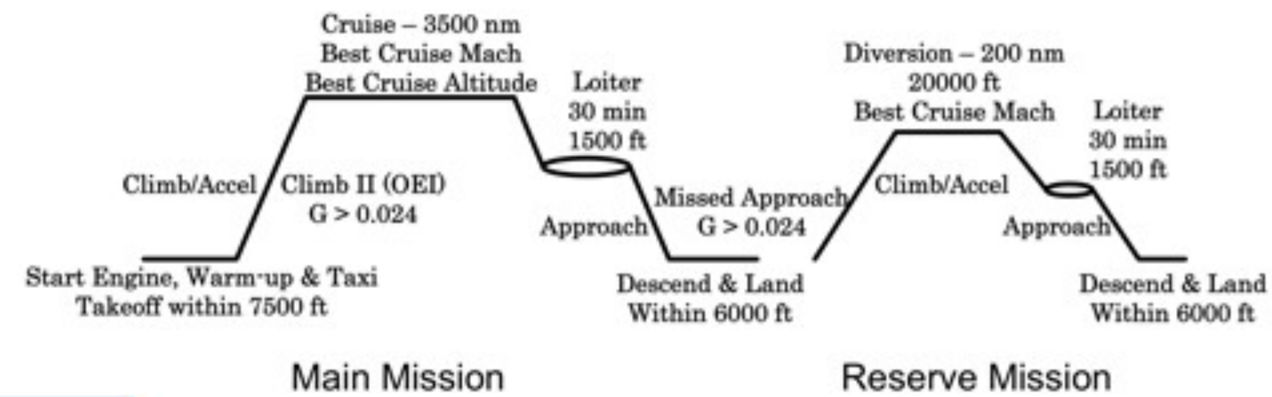
- No subtractive cancellation
- Numerically exact for small enough step

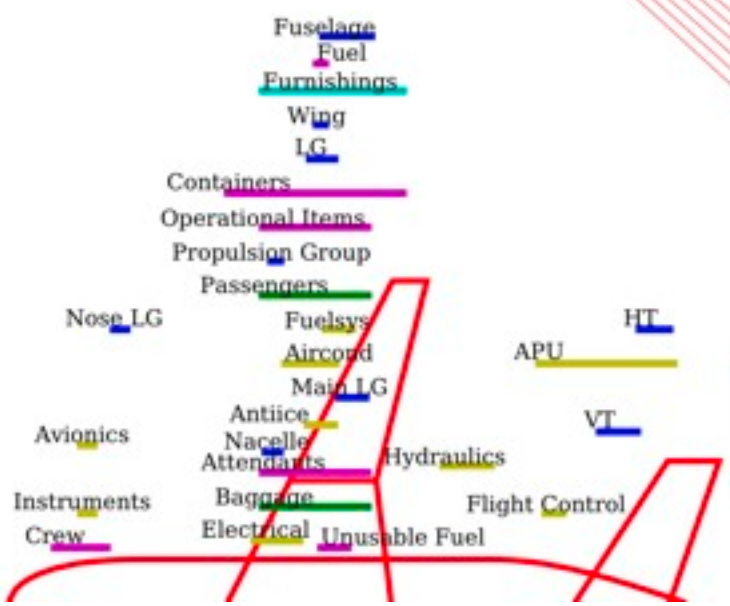
[Martins, Alonso and Sturdza, *ACM TOMS*, 2003]

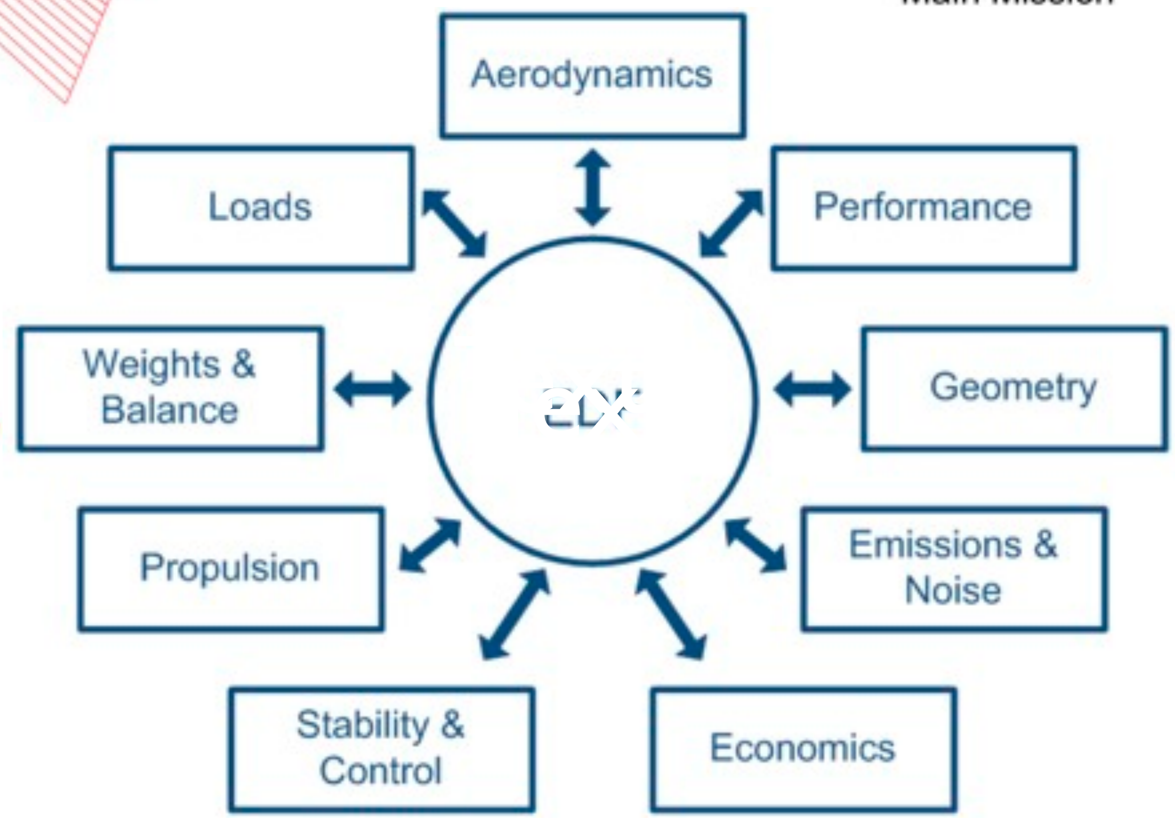
Aircraft Design for Minimum Environmental Impact

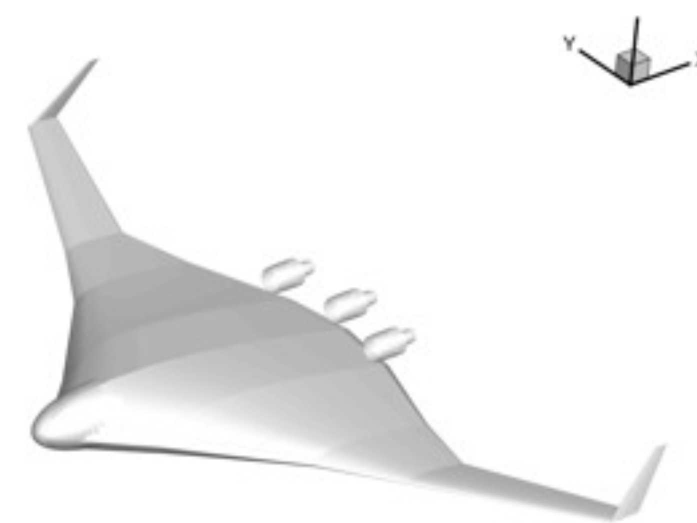
(Henderson, Perez, Martins, 2009)

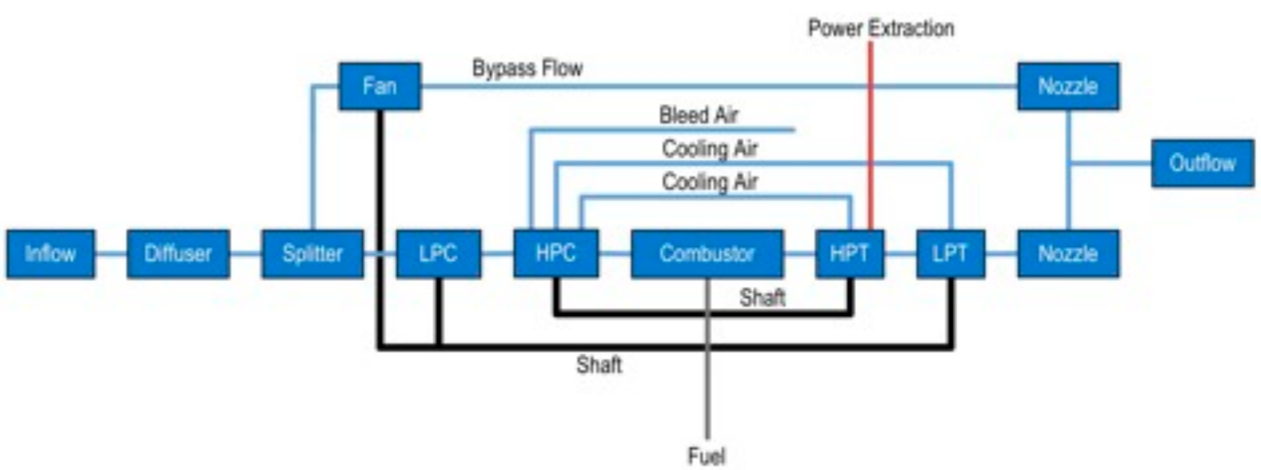


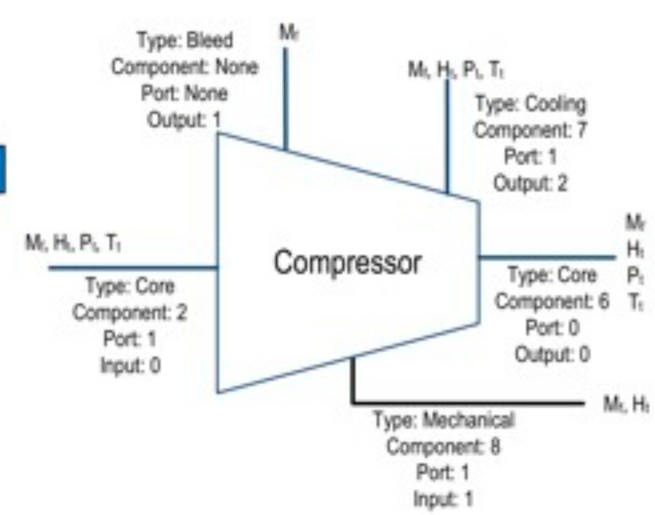


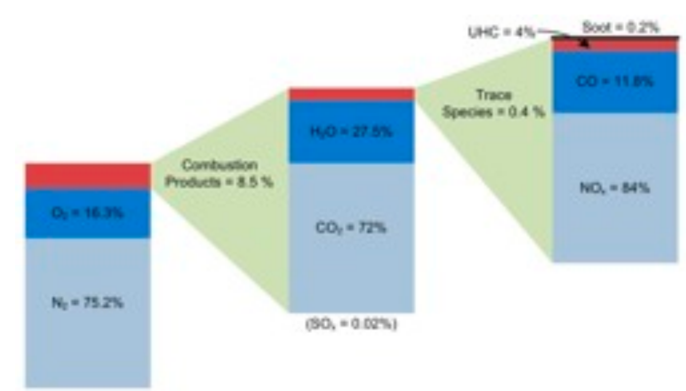




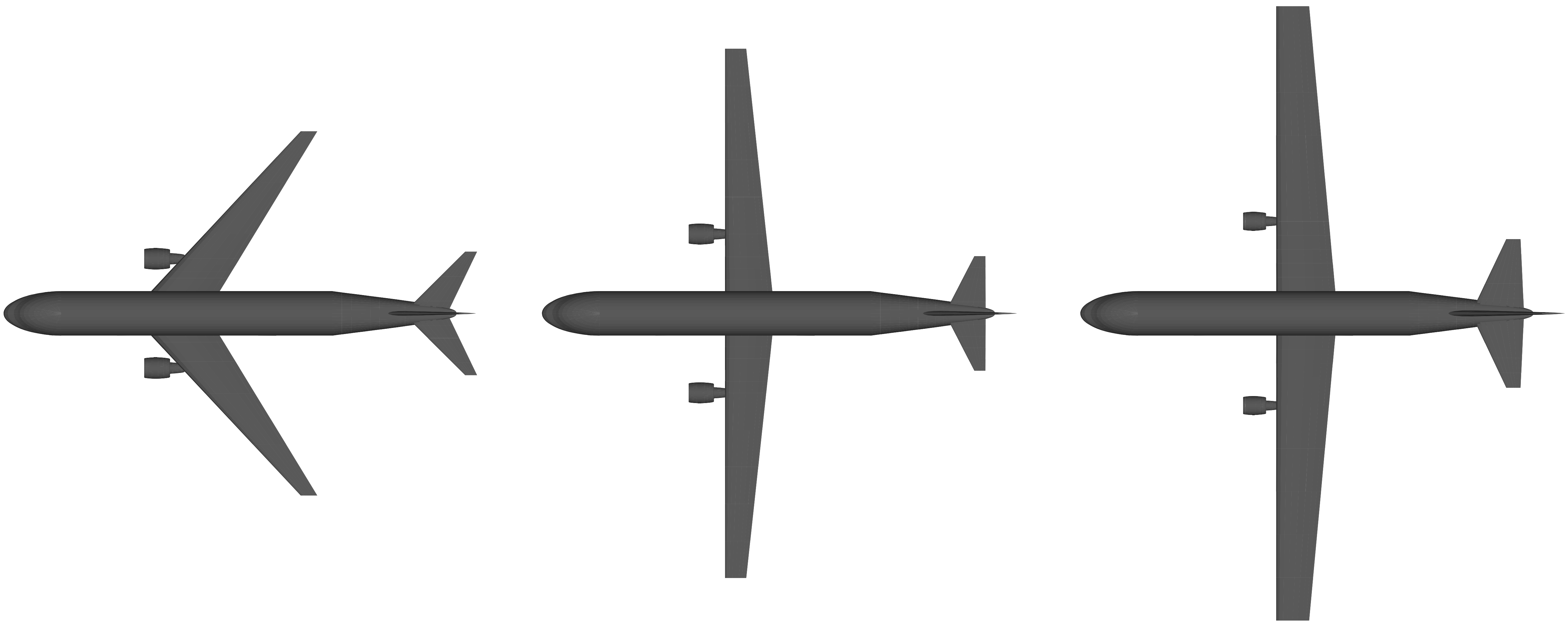






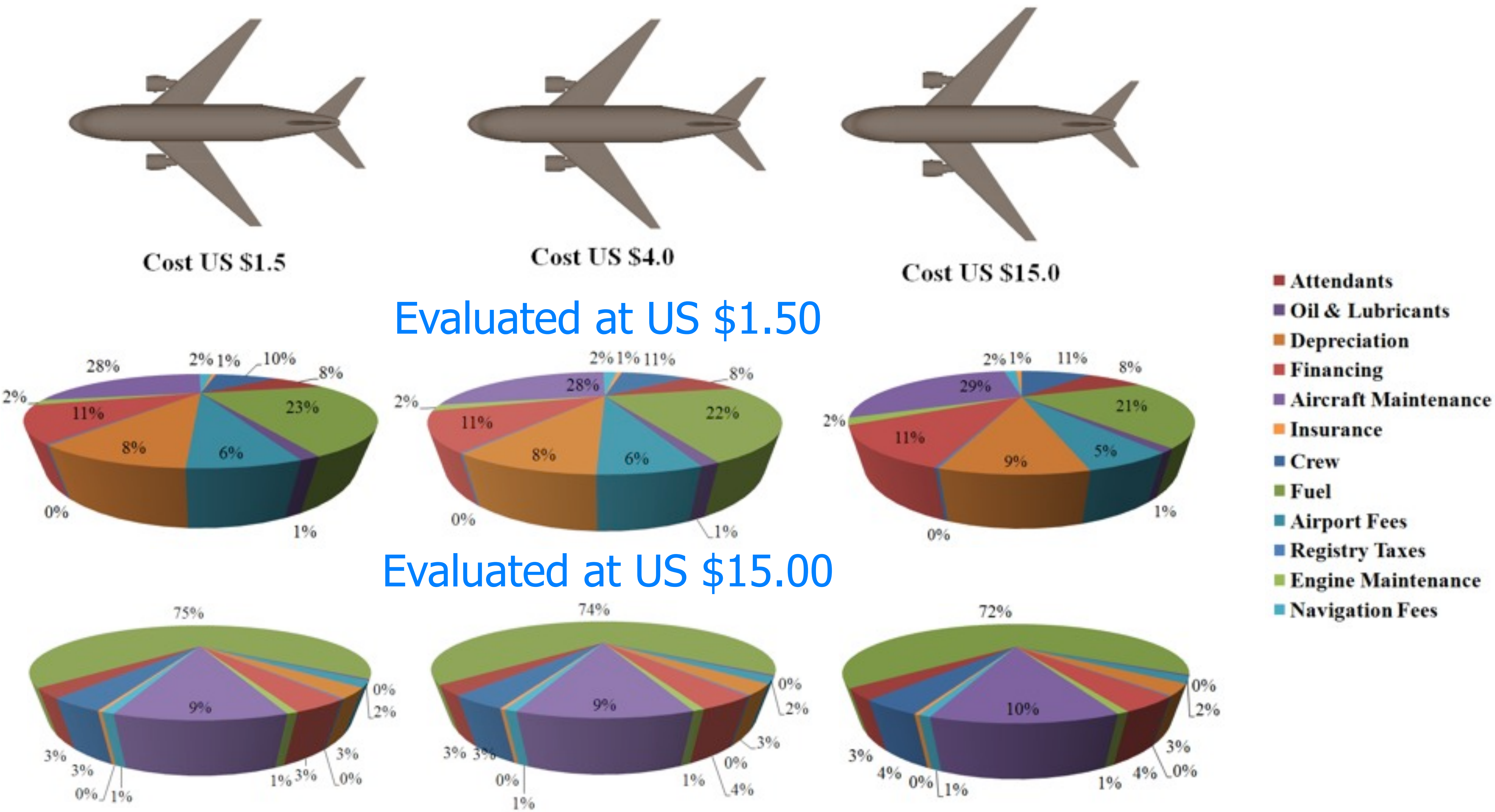


Single Objective Optimization

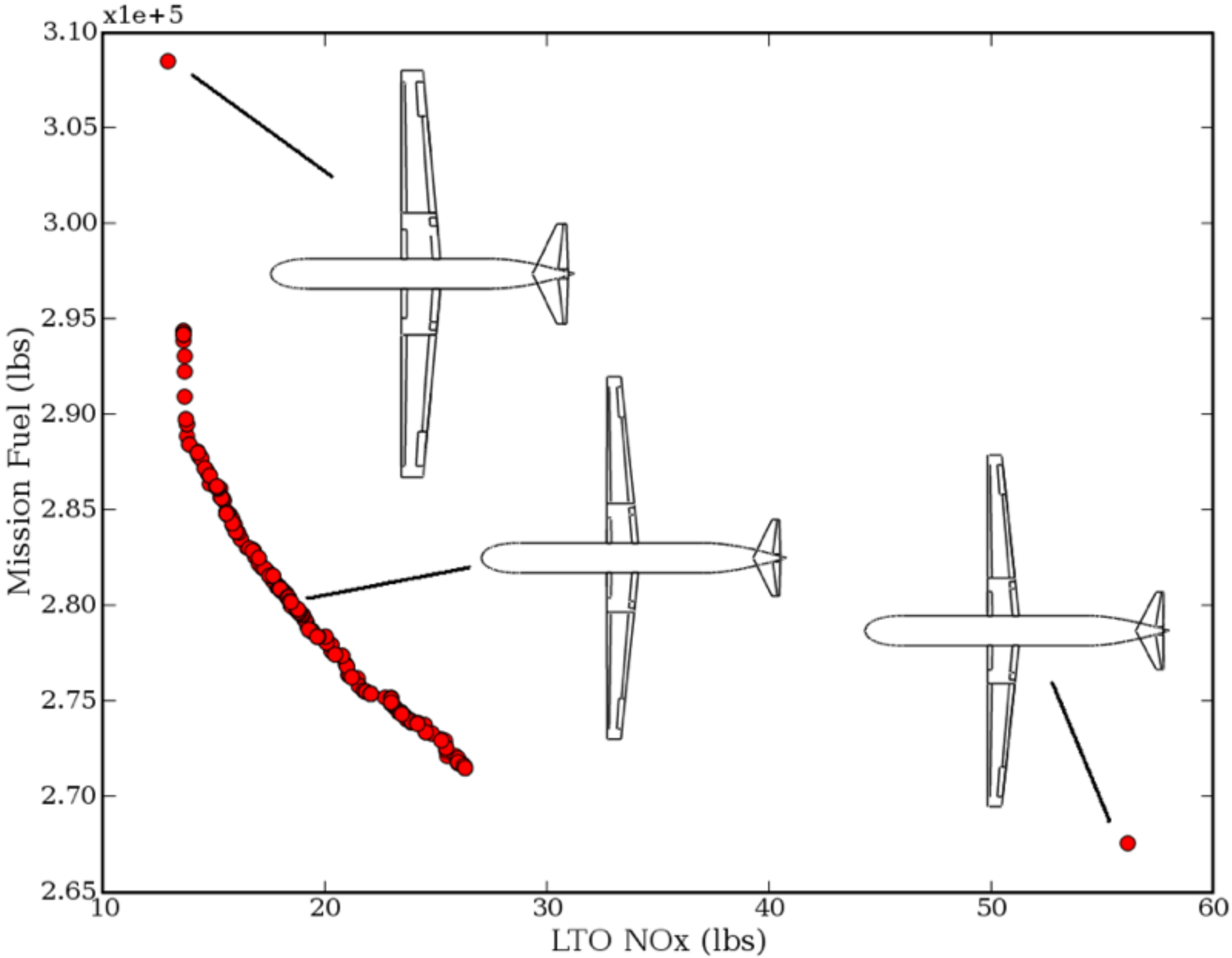


Cost Fuel Burn LTO NOx

Results for Increasing Fuel Prices



Multi-Objective Optimization



Wind Turbine Blade Design Optimization

(Kenway and Martins, 2008)

Wind Turbine Blade Design Optimization

(Kenway and Martins, 2008)