

Resch tessellations as stiff, folded modular shells

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Abstract

We present a method for designing and assembling modular, folded lattices based on numerically calculated triangular Ron Resch molecules. These lattices form stiff structural grids of beams capable of realizing both single- and double-curvature surfaces. Our work builds upon Ron Resch's contributions to topology and geometry, where he first introduced folded tessellations as structural systems. His work laid the foundation for major advancements in architecture, structural engineering, and computational design, establishing him as a key figure in the emergence of Structural Origami as a distinct field. While much research has focused on the kinematic aspects of Resch's work, we find a gap in its less explored potential for static structural applications. First, we describe how to compute triangular Ron Resch tripods to form a stiff and constrained unit cell, which we then assemble into a lattice. The resulting structure forms a woven hexagonal grid of beams with continuous second moment of inertia across the assembly, contributing to high stiffness. Next, we demonstrate how to numerically calculate developable unit cells to approximate target surfaces with single or double curvature. We believe this method—where global geometry and mechanical performance are encoded within the local crease pattern—offers significant potential for scalable structural folded shells, with applications from engineering to architecture.

Keywords

Structural origami, freeform tessellations, Resch patterns, modular shells, discrete assembly

Introduction

Origami modules in architectural geometry

Architectural geometry offers various methods for achieving doubly curved surfaces for architectural purposes by decomposition of surfaces into simple pieces to be assembled into the target shape. Surfaces can be decomposed into planar quadrilaterals (PQ)¹ or the strips of singly curved sheets² using conjugate net and principal curvature lines. The supporting structures of surfaces have been simplified using torsion-free beams¹ along the principal curvature lines. Recent studies construct structures through the bending of straight slats along the geodesics or asymptotics.³ Origami, the art of sheet folding without stretching the material, is a powerful way to create complex modules that serve as both surface and structural elements of freeform geometry from a simple manufacturing process.^{4,5} In this paper, we derive our inspiration from the work of Resch to generalize his patterns and combine them to make

a stiff and lightweight structural element that can suit a given doubly-curved surface.

Resch's folded tessellations and research impact

Ronald Dale Resch's research on computational design, geometry, and digital fabrication has had a lasting impact. He pioneered the architectural use of folded tessellations,

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developed early parametric CAD tools, FEM simulations, and custom CAD-CAM systems, laying the groundwork for today's digital design workflows.⁶

Resch's origami-based work began in the 1960s and was documented in his self-produced film *Paper and Stick Thing*.⁷ In 1970, Resch and Christiansen proposed folded tessellations as transformable structural systems, combining geometric design with elastic and kinematic analysis for simulation.⁸ Resch later expanded this framework, defining structural lattices as topological generating systems with two rules: one for the geometry of parts and another for their assembly.⁹

The geometric flexibility of Resch cells was early recognized and extensively applied in research. They have been proposed as mechanical metamaterials,¹⁰ energy absorption structures,¹¹ sandwich panels,¹² cellular materials,¹³ and scaffolding for membrane tensegrity shells,¹⁴ among other studies. More similar to our contribution, the most notable previous work is the development of free-form Resch-like tessellations by Tachi,¹⁵ as a continuation of his work Origamizer and Freeform Origami, where he shows how this family of lattices are generalizable to any polyhedral surface while also showing control of avoiding self-intersections, making it more suitable for folding materials with thickness or that cannot satisfy 180 degree folds.

Cellular and architected materials

Origami tessellations and cellular materials share a modular nature. We examine recent work in architected cellular materials in the macro scale, both open-cell and closed-cell, to explore how assembly strategies influence mechanical performance. In particular, we focus on discrete assembly approaches, which overcome size limitations by building large structures from smaller components.

In open-cell, beam-based materials, this strategy has enabled ultralight, ultrastiff systems with record stiffness-to-density ratios,^{16,17} as well as tunable properties like chirality, auxeticity, and compliance.¹⁸

In closed-cell materials, origami has allowed the design and fabrication of doubly curved structural shells from foldable modules.^{5,19–22} Structurally, plate-based materials can outperform beam-based counterparts at equal relative density. As shown by Tancogne-Dejean et al.,²³ for equal solid volume under uniaxial loading, two plates carry the load instead of one beam, doubling the strain energy stored at a given macroscopic strain and, consequently, the apparent stiffness. More broadly, closed-cell sheet topologies engage multiple load paths through membrane stresses, enabling them to approach the Hashin–Shtrikman bounds on isotropic stiffness.²⁴

Our work lies at the intersection of these two families, producing open-cell beam networks whose members are constructed from closed-cell folded plates, an open-cell

topology at the scale of the lattice, closed-cell plate mechanics at the scale of the member.

Our contribution

Given this context, we present a method for designing and assembling modular structural origami lattice shells. Each unit cell consists of two numerically computed, Resch-like triangular elements that, once assembled, mutually block their degrees of freedom (Figure 1). These cells are then discretely assembled into a lattice following a consistent topological strategy. The resulting structure forms a hexagonal grid of hollow beams with valence-3 interior vertices, which behaves under load as a woven network maintaining second moment of inertia continuously across the assembly. This produces a stiff structural system using only two material layers. The modular nature of the assembly offers flexibility for complex geometries, as the numerical approximation must satisfy developability only locally, at the level of each cell, rather than globally across the structure (Figure 2).

The surface approximation workflow maps a flat hexagonal grid onto the UV domain of a target surface and recovers local developability per cell through numerical optimization. This approach is effective for surfaces of low absolute Gaussian curvature, where the mapping preserves sufficient hexagonal regularity for the solver to converge. Strongly curved or topologically complex surfaces require an alternative discretization-first approach, which we identify as the primary direction for future work.

Compared to other structural origami methods (modular or continuum), our system presents a distinct combination of advantages: First, the inherent complexity of each unit cell's folding angles is automatically resolved by the assembly method. As there is only one configuration in which each calculated structure can exist, the assembly method uses the compliance of the tripods to adjust its correct folding state naturally. This makes the system highly suitable for on-site fabrication. Second, our approach achieves rigidity with only two material layers, in contrast to shown origami-inspired structures that require three layers to fully block cell DOFs. Third, this advantage is directly related to the preservation of the second moment of inertia. Our method creates a continuous structural volume where all nodes are interconnected. This not only improves structural performance but also provides space for additional functions, like insulation, heat exchange, or routing of electrical systems. The same interconnected volume can also serve as lost formwork, enabling the casting of materials like expanded foams, concrete, or clay. Furthermore, openings between molecules can be specifically designed to address natural ventilation and lighting.

We believe this method offers a valuable contribution to the development of structural folded shells across scales. The discrete, modular approach enables scalability, making

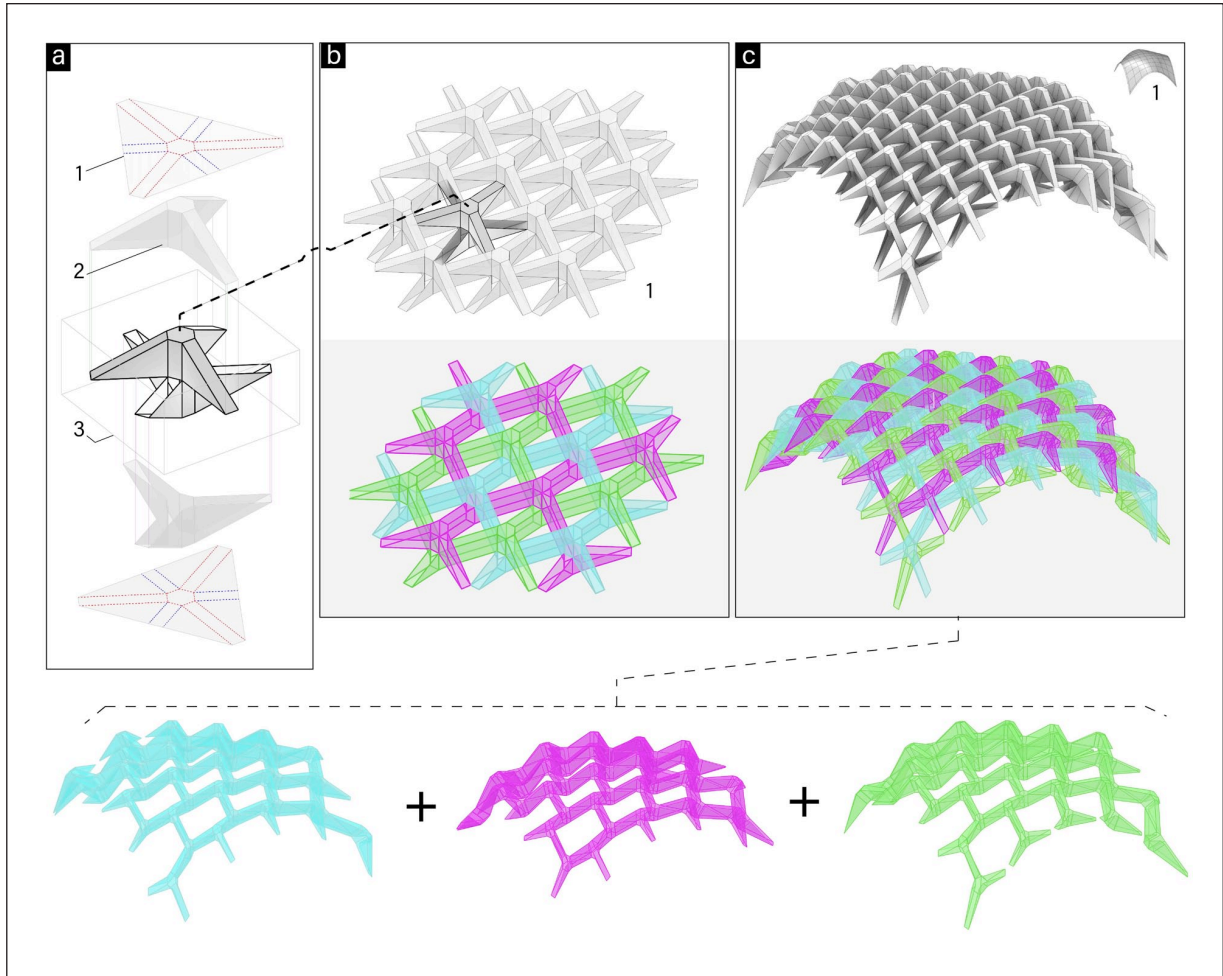


Figure 1. (a) Unit cell assembly. 1—Triangular Resch unfolded cell. 2—Folded. 3—Two tripods form a unit cell, (b) unit cell assembly into a planar lattice, (c) planar lattice is calculated to satisfy a target surface. 1—target surface. Gray highlighted area shows the virtual hexagonal grid of woven beams in its flat state and in its shaped state. For the rest of the paper, each hexagonal group is highlighted in Blue, Magenta, and Green.

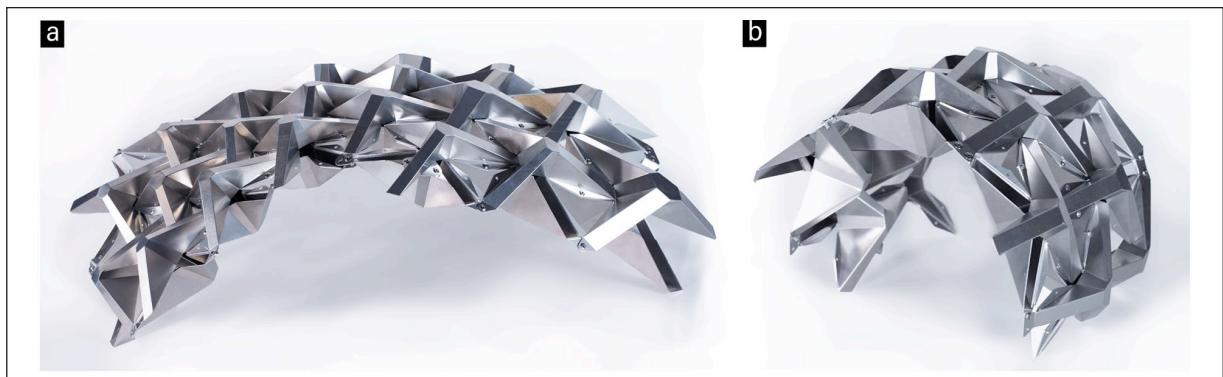


Figure 2. Manufactured aluminum models for this paper: (a) doubly curved woven beams and (b) single curvature woven beams.

the system suitable not only for architectural applications but also for broader engineering and robotic use cases, from micro to macro scale.

The structure of the paper is as follows: first, Section *Regular Geometry* shows how we calculate and connect regular triangular Resch-like modules into a lattice and the

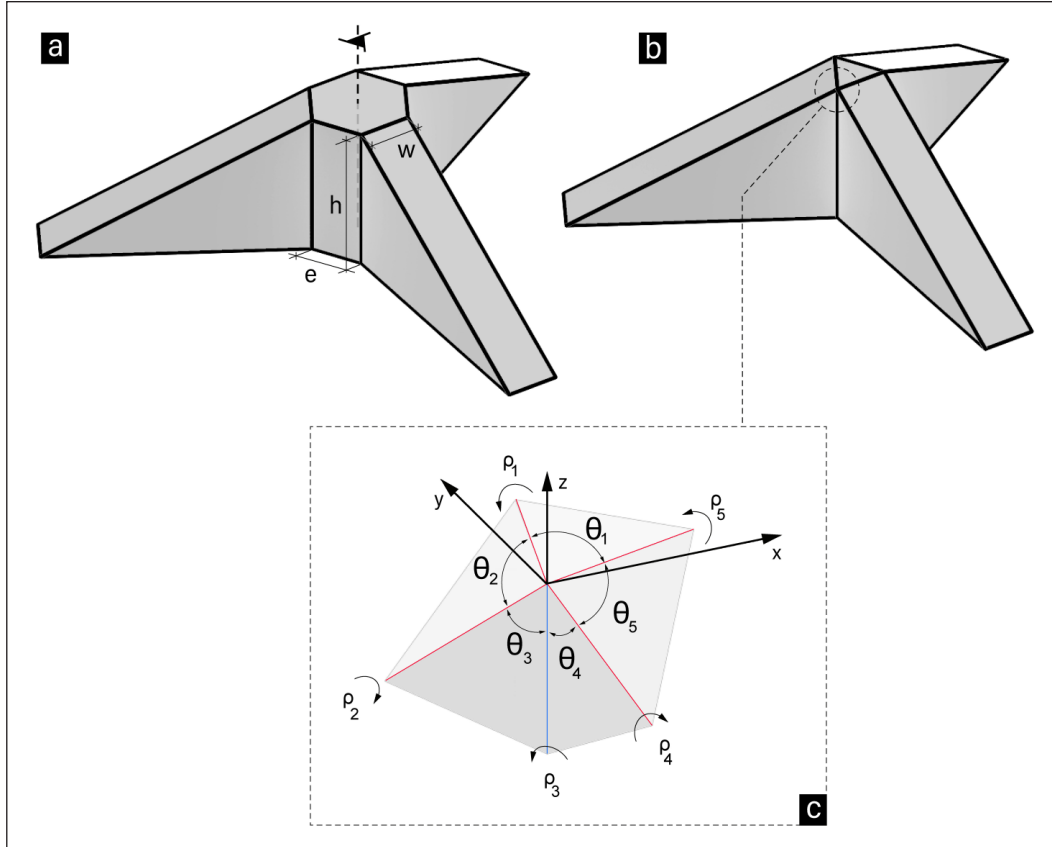


Figure 3. Unit cells DOFs, represented as e , h , w , ρ_1 , θ_2 : (a) parametrized Ron Resch triangular cell, (b) simplified version for vertex analysis, and (c) detailed vertex analysis: crease assignment, angles, and rotations.

structural advantages of this method. Then, in Section *Freeform Surface Approximation* we present the numerical approach for designing the geometry of each unit cell to ensure that the final structure matches a target surface provided by the user. Finally, we show the results to validate our proposed method, concluding with reflections and future work.

Regular geometry

The geometry of woven beams

We first derive the flat grid analytically, taking advantage of the regular cell's symmetry to obtain a closed-form solution. This serves as the starting configuration for the numerical optimization that computes each unit cell geometry for a target surface.

Analytical solution of regular cells. To analytically determine an arbitrary folding state of a regular Resch triangular cell, we first establish its parametrization. The cell has three freely chosen geometric design parameters: e , the expansion magnitude along the central valley crease; w , the cell width; and h , the cell height. These define the proportions of the flat state. The crease angles θ_1 through θ_5 are fully

determined by the regularity condition given in equation (1). With e , w , and h fixed and all crease angles set by regularity, the complete folding motion of the cell is governed by a single kinematic degree of freedom, ρ_1 (notation is shown in Figure 3(a)). To solve for the folding kinematics analytically, the unit cell is simplified by removing the expansion segment e and analyzing the resulting degree-5 vertex, as shown in Figure 3(b) and previously demonstrated in Parra Rubio et al.²⁵ This reduction makes the kinematic problem tractable in closed form. The goal of the analytical solution is to express every intermediate folding state of the cell as a closed-form function of ρ_1 alone. Figure 3(c) shows the parametrization of the degree-5 vertex, indicating the crease angles θ , rotation angles ρ , and the mountain-valley assignment.

We calculate that our unit cell has five constraints. We select them to be e , w , h , ρ_1 and θ_2 .

As we work with regular Resch cells, all crease angles are set as follows:

$$\theta_1 = \theta_3 = \theta_4 = \frac{\pi}{3}, \theta_2 = \theta_5 = \frac{\pi}{2} \quad (1)$$

Knowing now all crease angles, and imposing values for e , w , h , we can calculate all intermediate folded states as a

function of a single parameter ρ_1 . For that, we now define vectors corresponding to each crease, where each vector is associated with a rotation angle ρ . Let $x_1, \dots, x_5$ denote the vectors aligned with the creases corresponding to rotation angles $\rho_1, \dots, \rho_5$, respectively. Using the coordinate system shown in Figure 3(c), we define:

$$x_1 = (\sin(5\pi/12), \cos(5\pi/12), 0) \quad (2)$$

$$x_5 = (\cos(5\pi/12), \sin(5\pi/12), 0) \quad (3)$$

The positions of x_2 and x_4 are linked by symmetry, as they lie in the plane $x = y$. To compute x_2 , we multiply two rotation matrices: the first rotates x_1 about the Z-axis by an angle θ_2 , and the second rotates the resulting vector about the axis x_1 by the angle ρ_1 .

The resulting vector positions are:

$$x_2 = \left(-\frac{(\sqrt{2} + \sqrt{6})\cos\rho_1}{4}, \frac{(-\sqrt{2} + \sqrt{6})\cos\rho_1}{4}, -\sin\rho_1 \right) \quad (4)$$

$$x_4 = \left(-\frac{(-\sqrt{2} + \sqrt{6})\cos\rho_1}{4}, \frac{(\sqrt{2} + \sqrt{6})\cos\rho_1}{4}, -\sin\rho_1 \right) \quad (5)$$

Following the approach in a previous study,²⁵ x_3 is defined as a unit vector lying in the plane $x = y$. For simplicity, we parametrize it using a scalar a . Then, x_3 is expressed as:

$$x_3 = a \hat{x} + a \hat{y} - \sqrt{1 - 2a^2} \hat{z} \quad (6)$$

Since all crease angles θ are known, the solution for a takes a quadratic form. Geometrically, this corresponds to the intersection of a circle with a plane, giving two possible solutions. We select the solution consistent with the prescribed mountain–valley assignment:

$$a = \frac{\sqrt{2}(u_1 - u_2)}{u_3} \quad (7)$$

$$u_1 = 2\sqrt{3} \cdot \sqrt{\sin^2(\rho_1)} \cdot \sin(\rho_1) \quad (8)$$

$$u_2 = \cos(\rho_1) \quad (9)$$

$$u_3 = 2(3\sin^2(\rho_1) + 1) \quad (10)$$

The topology of woven beams

With the tripod geometry established, we now describe the topological assembly rule that governs the assembly of our lattice.

From cell to lattice. Figure 4 shows the nature of our structural system as a tessellated 3D lattice, composed of unit cells formed by assembling two triangular Ron Resch cells in a specific configuration. Two key aspects of this topological assembly rule contribute to the high stiffness of the resulting structure.

First, we examine the stiffness of the unit cell itself. We show in Figure 4a, that the formation involves three steps: starting with a triangular Resch cell (1), mirroring it across the base plane (2), and then rotating it by $\frac{\pi}{3}$ around the

axis normal to the mirror plane. This final step constrains the cell's degrees of freedom. While the intermediate configuration in Figure 4(a)-2 exhibits bi-stability with infinitesimal flexibility, the final configuration in a-3 is fully rigid, as the deformation potential of one cell is blocked by its mirrored counterpart.

Second, we study the stiffness of the tessellated lattice. By applying translational symmetry using the vectors shown in Figure 4(a)-3, strong inter-cell connections are formed. This assembly produces continuous beams a constant second moment of inertia across the two cells. Figure 4(b) highlights the constant cross section generated by this assembly and how these beams interlock to create a grid of woven beams.

The structural advantage of this topological assembly rule is demonstrated in Figure 5 for a theoretical flat configuration. Two models with the same unit cell but different assembly rules were simulated under a three-point bending test using ABAQUS CAE. Both models share identical conditions: a 3 by 7 cell array (275 mm long, 190 mm wide, and 27.4 mm tall) made of 0.5 mm thick aluminum sheet, a mass of 160 g, a central load of 30 N, and a support separation of 200 mm. Simulations show that, at equal mass and area, the woven beam configuration is up to 3.5 times stiffer than a tessellation formed from the configuration in Figure 4(a)-2. We highlight in blue the cross sections of both configurations.

Continuity of the second moment of inertia is necessary for this stiffness but does not fully explain it. Two further mechanisms contribute: one is moment continuity, as the woven cross section passes uninterrupted through each valence-3 vertex (Figure 4(b)) and transmits bending moment and shear across the node. The second one is tri-axial redistribution, as three continuous beam directions meet at every interior vertex, spreading load across the array rather than concentrating it along a single line of members.

To locate the performance of this work relative to the state of the art in architected materials, Figure 5(c) presents a qualitative comparison. Relative stiffness is chosen because of both convention and data availability. Strength is widely reported in absolute terms, but normalizing it requires a constituent yield strength that is rarely stated and ill-defined for laminated constituents. The plot places

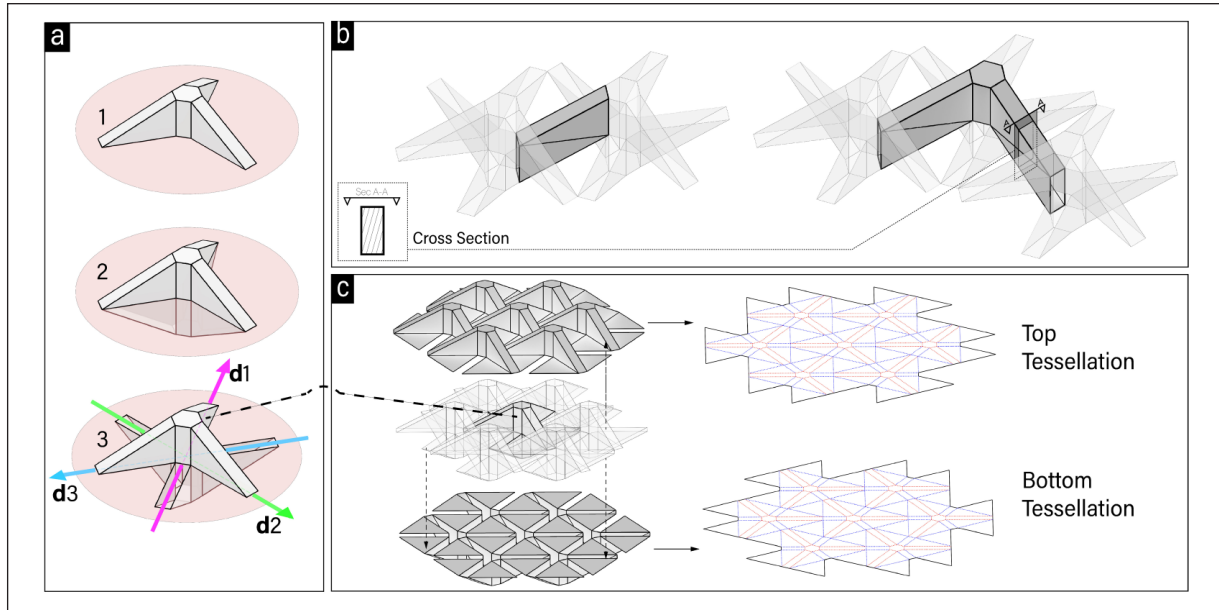


Figure 4. (a) Three steps to form a unit cell. In the final step, lattice translation vectors are shown, (b) detail of the formed beam after translational symmetry, and (c) a tessellated unit cell forms a lattice of woven beams, which can also be seen as two regular triangular Resch tessellations—one top and one bottom—assembled together.

the system to within an order of magnitude, which is the resolution this comparison supports.

Lattice discretization: Global versus local developability. Figure 4(c) shows how the proposed lattice structure can be understood as the combination of two origami tessellations: one forming the upper layer and another forming the bottom layer. The original triangular tessellation introduced by Resch⁹ is globally developable, meaning it forms multiple triangular unit cells from a single continuous sheet with a defined mountain–valley pattern. While work by Tachi shows that this pattern can be numerically calculated to satisfy arbitrary surfaces,¹⁵ the resulting crease map becomes highly complex.

From a manufacturing standpoint, origami tessellations with global developability behave like parallel mechanisms, requiring all creases to be formed simultaneously and following its rigid motion, a manufacturing challenge that remains unsolved for load-bearing materials at scale.

To address these challenges, we apply principles from discretely assembled materials, where unit cells are assembled into lattices that exhibit structurally continuous behavior at the macroscale. By discretizing both the upper and lower tessellations, we enable scalability of the manufacturing process to architectural dimensions, as the primary size and complexity constraint becomes that of the unit cell. Our approach, based on local developability, reduces pattern complexity and relaxes constraints in the numerical optimization, allowing for the realization of complex surfaces through simpler unit cell geometries.

Manufacturability. Due to the modular nature of our system, additional features are required to connect adjacent cells and ensure manufacturability at architectural scales. The connection system is central to this method, as it is what gives the structure its discretely assembled nature. We show in Figure 6 the two connection strategies proposed in this work. Both approaches introduce an extra pleat at each cell leg, but the folding process remains trivial. Furthermore, by connecting modules through surfaces rather than edges, these methods enable accessible and reversible assembly techniques, such as sheet metal screws, inserted nuts and threaded fasteners, or blind rivets.

The first strategy follows the natural woven pattern, as shown in Figure 6(a). In this approach, pleats belonging to the same hexagonal subdivision (highlighted in the same color) are assembled together. This method is particularly suitable for larger structures with a large number of modules, as it also minimizes the number of components needed.

On the other hand, a simpler connection method is shown in Figure 6(b), which requires additional triangular joining pieces. Due to the triaxial nature of the lattice, the gaps between neighboring cells naturally form planar triangles already visible in the global developable pattern in Figure 4(c). These triangular connectors allow joining adjacent cells, even if they belong to different hexagonal subdivisions. Beyond structural purposes, this method can also provide architectural benefits such as insulation or controlled light entry.

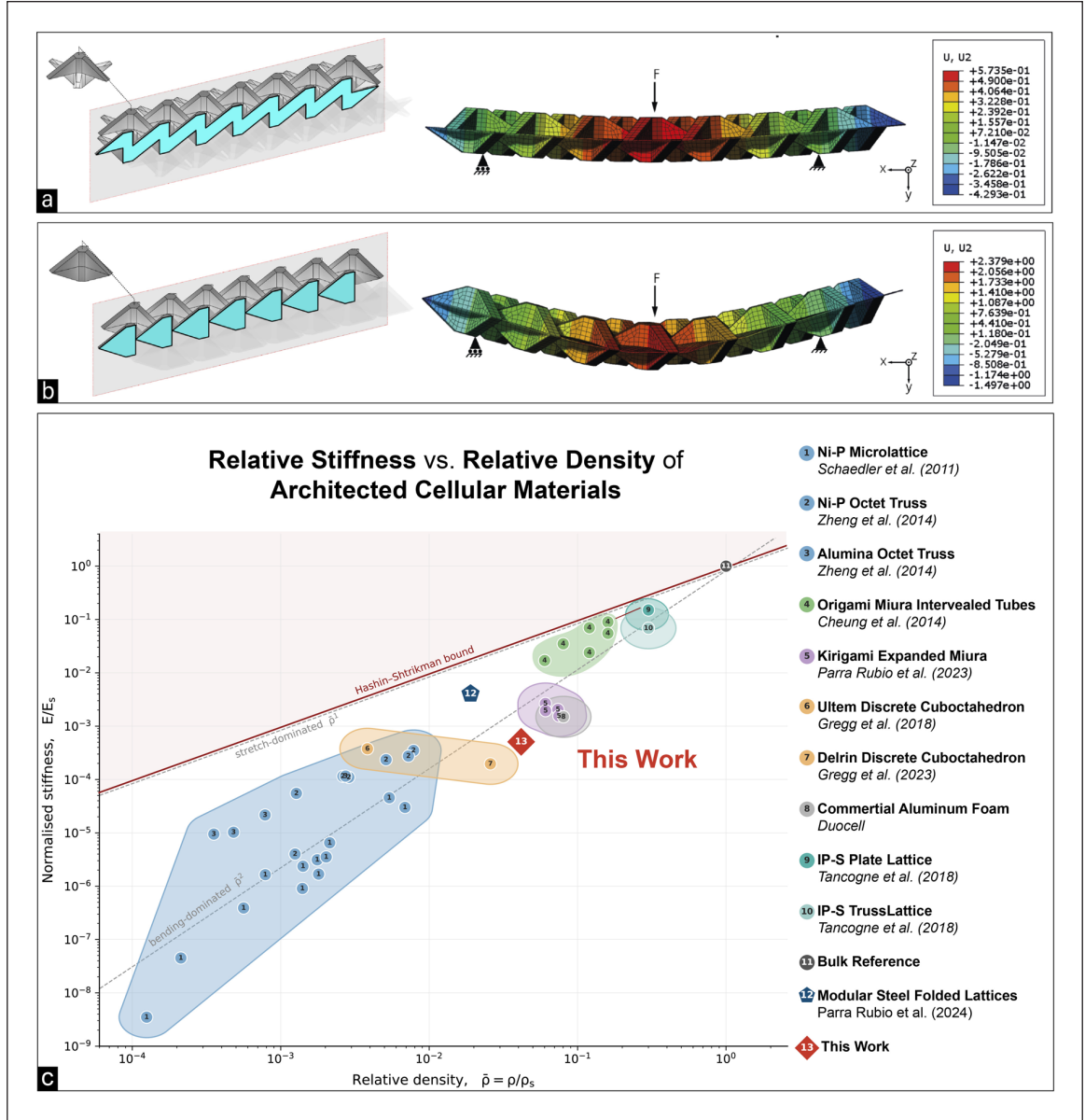


Figure 5. (a) and (b) three-point bending test for the two commented configurations. Colormap shows displacement at the same magnification in the Y axis. Units in mm. (c) Normalized stiffness versus relative density for architected cellular materials: octet lattices,^{26,27} Miura-ori interleaved tubes,²⁸ kirigami plate lattices,²⁹ modular steel lattices²⁵ cuboctahedral lattices,^{30,31} stochastic foam, and plate and truss lattices.²³ Lines are the Hashin–Shtrikman bound,^{24,32} the isotropic truss ceiling, and bending-dominated scaling.³³ This work's value (13) is a lower bound, derived from flexural rigidity at $L/h = 7.3$. The comparison is qualitative.

Freeform surface approximation

Numerical approximation for surface satisfaction

We have shown that the flat tessellation can be designed analytically fairly easily due to its multiple axes of symmetry and known constraints; the same cannot be said when the tessellation approximates a single or doubly curved surface. In fact, depending on the applied constraints, and in order to preserve the developability of each

molecule, the resulting shape of the molecules themselves can change drastically, eventually together with the analytical formulation that would be needed to design it.

The first non-trivial challenge is finding the degrees of freedom that characterize the design space of each single molecule when distributed onto a non-flat surface. For example, can the quad faces of the molecule be forced to be planar? Is it possible to change the height of the molecule arbitrarily? Is it possible to force the top face to be symmetric with respect to the three arms? Can the top face be forced to be a regular polygon? Can we apply multiple

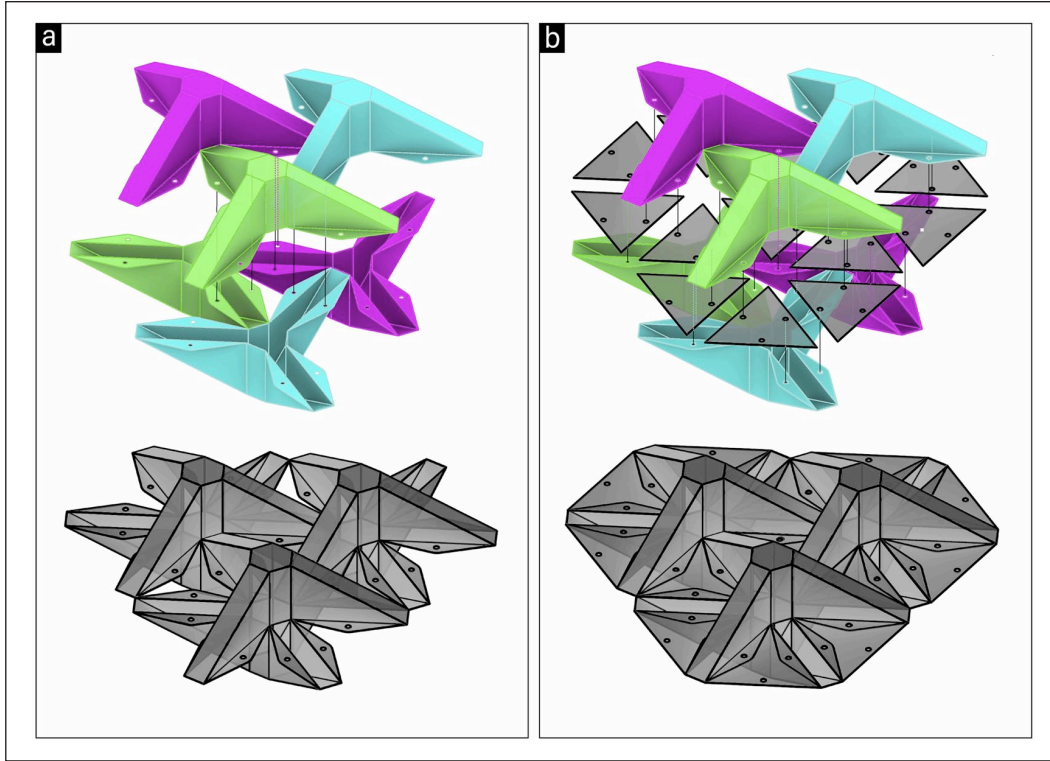


Figure 6. (a) Assembly using the hexagonal subdivision method and (b) assembly using an additional triangular piece.

of these constraints at the same time, or are some combinations incompatible with the molecule's developability?

The second challenge is designing an appealing shape while complying with all the imposed constraints, and doing it analytically would be much longer and harder because it would require adjusting the formulation every time there is a need to change the input constraints.

Thus, we developed an iterative approach based on numerical approximation and error minimization, which enables us to experiment dynamically and creatively with different constraints and observe empirically when the various combinations of constraints converge to a smaller or bigger developability error (refer to Figure 7).

The error minimization is based on relaxation methods, implemented using combining different Goals, that is, the objective functions, in Kangaroo solver in Grasshopper for Rhino8,^{35–37} which requires as input a series of particles (the vertices of a mesh) and allows the application of constraints in the form of vector forces that are applied following specific rules.

The Kangaroo solver iteratively minimizes the total sum of the weighted squares of all the move distances set by some GoalNodes which set some specific constraints, by following this rule:

$$P_{i,new} = P_{i,cur} + \frac{\sum_{j=1}^n \omega_j G_j}{\sum_{j=1}^n \omega_j} \quad (11)$$

where i is the index of the vertex/particle P , n is the number of goals acting on that vertex/particle, ω is the weighting/strength (assigned via Grasshopper's nodes), and G is the moving vector exerted by the constraint/GoalNode j .³⁸ Detailed formulations of each Kangaroo's GoalNode,³⁵ and the full Grasshopper algorithm for molecule optimization³⁴ are openly available on GitHub.

Developability enforcement general approaches

In general, it is possible to enforce the developability of a generic 3D triangulated mesh by equalizing the lengths of its edges with those of its elastically relaxed 2D pattern, with no cuts and overlappings, which shares with the 3D mesh the same topology. In Grasshopper and Kangaroo, this method was previously tested by the authors³⁹ and proved to be more robust and stable compared to the alternative method based on enforcing the angles at each vertex to sum up to 360°, nevertheless, it is limited to triangulated meshes.

The simplest developability enforcement case is when the lengths of the edges of a 2D developable pattern are sent unilaterally to the edges of a 3D non-developable shape, which shares with the 2D pattern the same topology; however this unidirectional enforcement will change the 3D configuration in a very strict, and sometimes unpredictable way. Thus, in cases where the 3D shape must comply with additional constraints (e.g., target anchor points, symmetry) while being developabilized, the

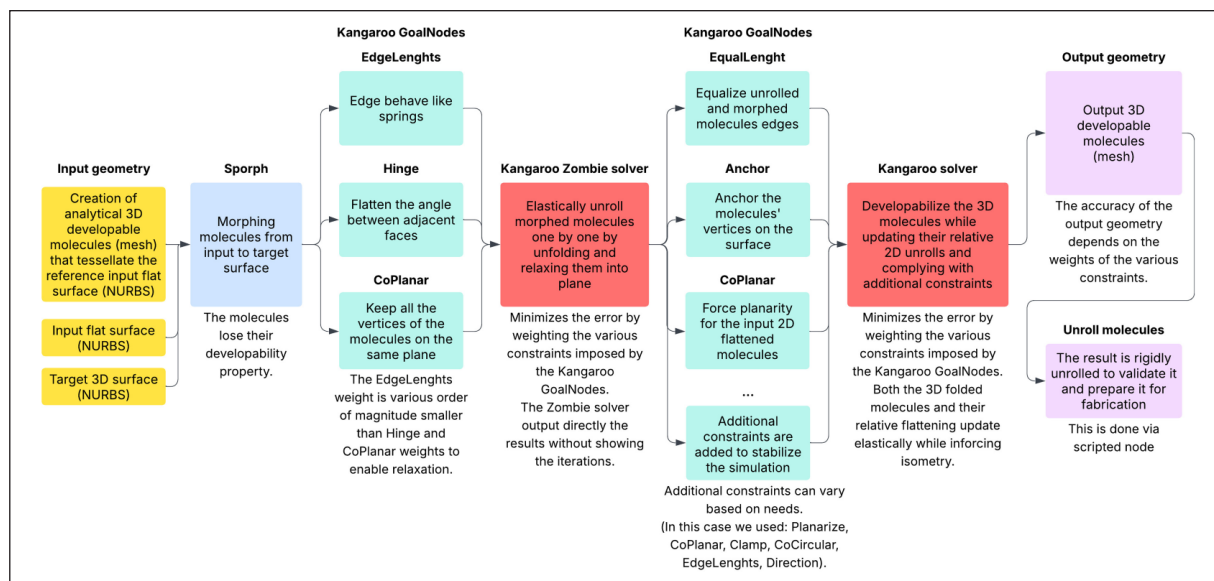


Figure 7. Schematic synthesis of the Grasshopper algorithm. The full Grasshopper definition is available on GitHub³⁴

unidirectional length equalization is too limited. In these latter cases, it is preferable to simultaneously update the lengths of the edges of both the 3D shape and its relative 2D flattened version for each iteration; in this way, both the 3D and the flat pattern change simultaneously, allowing them to comply with additional constraints both in 2D and 3D.

In order to do so in Kangaroo, it is sufficient to use the EqualLength Goal node to equalize the lengths of the edges of the 3D shape and its relative in the 3D pattern, while keeping the pattern planar with the CoPlanar Goal node (or OnPlane node). With this method, it is fundamental to input some additional constraints to stabilize the simulation (e.g., anchor points).

Developability enforcement in the specific case of the Resch pattern

The method to conform the Resch pattern variation to a given surface is based on the simultaneous length enforcement described in the previous section, as follows.

First, analytically construct the 3D folded developable tripod and tessellate it over a planar surface, as described in the previous section. Next, use surface-to-surface morphing to transfer the planar tessellation from a singly curved source surface to a doubly curved target surface with the Sporph node in Grasshopper. This process remaps the vertices of the original planar tripods according to the u and v parameters and the local normal directions of the source and target surfaces, allowing the geometry to conform to the new surface. As a result, the deformed tripods are no longer developable.

The next step consists of applying forces to the tripods iteratively and deforming them until their developability property is restored while imposing some constraints (see below). The simulation stops when the average movement threshold falls below 1×10^{-15} units per time step. The number of iterations can vary significantly, and in some cases the system may fail to converge to equilibrium, depending on the chosen target surface and tripod distribution. When this happens, the weights of the GoalNodes or the solver threshold can be tweaked; alternatively, the input geometry can be manually adjusted until convergence is achieved. As a reference, a cylindrical surface patch with a radius of about 120 mm, 12 top tripods, and 12 bottom tripods with an outer edge of about 90 mm each, arranged in six columns and two rows covering the whole surface, converged after 4200 iterations, with a maximum isometry area error of 0.002% for a single tripod.

The first and most important property to constrain is the developability of the individual molecules, which is actuated via isometry enforcement between the flat pattern of each molecule and its relative 3D folded configuration (actuated via the "EqualLength" GoalNode). This constraint is responsible for preserving the property of the molecule of being flat-unfoldable on the plane, which is necessary for the fabricability of the structure from flat metal sheets. This constraint equalizes at the same time the lengths of the corresponding edges of the flat and the folded meshes. In order for this to work, both meshes need to be triangulated to prevent shearing deformation of polygonal faces, and the vertices of the flat-unfolded mesh must be constrained to stay coplanar (via "CoPlanar" GoalNode).

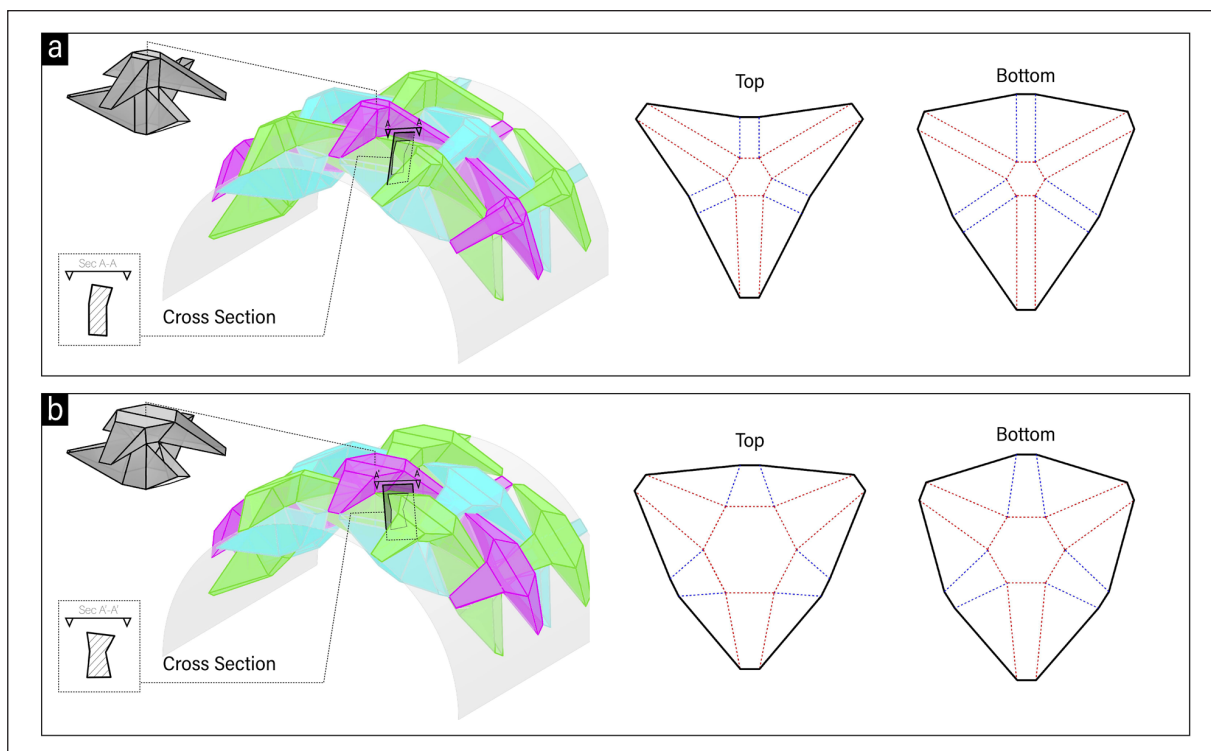


Figure 8. Two valid developable solutions with minimal developability error achieved by changing the clamping minimum and maximum edge lengths. (a) Solution with smaller allowed edge lengths (section A-A). (b) Solution with larger allowed edge lengths (section A'-A').

Another fundamental constraint is the identification of the anchor points, which must not be affected by the developability enforcement and other constraints. This is crucial to guarantee that the molecules will remain connected at specific points during the simulation. In our case, all the points in the perimeter of the molecule contacting the target surface were set as anchors.

These constraints alone are sufficient to make the simulation converge to an equilibrium point with minimal error. However, polygonal faces could bend due to the triangulation of the mesh, and the resulting adjusted mesh might not be symmetrical with respect to the anchor points and the principal curvature directions of the target surface, furthermore by grabbing and manually pulling the mesh vertices (via “Grab” GoalNode) other valid developable configurations can be explored smoothly and no local stable equilibrium states are observed. This means that the system is under-constrained. This could be useful in some cases, but we aimed at a more consistent and controlled shaping of all the molecules; thus, we experimented by adding additional constraints.

We added a constraint responsible for planarizing polygonal faces (through the “CoPlanar” GoalNode), and we limited the length range of each edge to improve its symmetry and shrink its overall configuration space. These two additional constraints enabled to reach a stable equilibrium; however multiple local equilibrium states with varying minimum developability errors were observed. These local equilibrium states could be accessed by

grabbing a vertex of the mesh and dragging it far enough to allow it to stabilize in a different configuration. When no other equilibrium state was found, the mesh bounced back to its initial stable configuration. The various local equilibrium configurations could be tweaked by varying the minimum and maximum edge lengths or the weight of each constraint. However, at the various equilibrium states, the error was not always minimized to zero, which could either mean that the system was over-constrained, or that the local minimum found was not the best one.

In order to find a configuration where the error was under a certain threshold, we had to tweak the input parameters and weights. The best set of parameters depends on the specific configuration and size of the molecule in relation to the target curved surface. We observed that molecules located in concave regions of the surface were particularly resistant to simultaneously minimizing their error with those positioned in convex regions. We also observed that different quadrilateral faces in the flat configuration tended to collapse into triangles, depending on whether they conformed to concave or convex surface regions.

Discussion on Resch pattern surface approximation and implication for fabrication

The first study model (Figure 8(a)) is based on a cylindrical surface, where molecules on the intrados and extrados are easily separated and optimized using distinct constraint

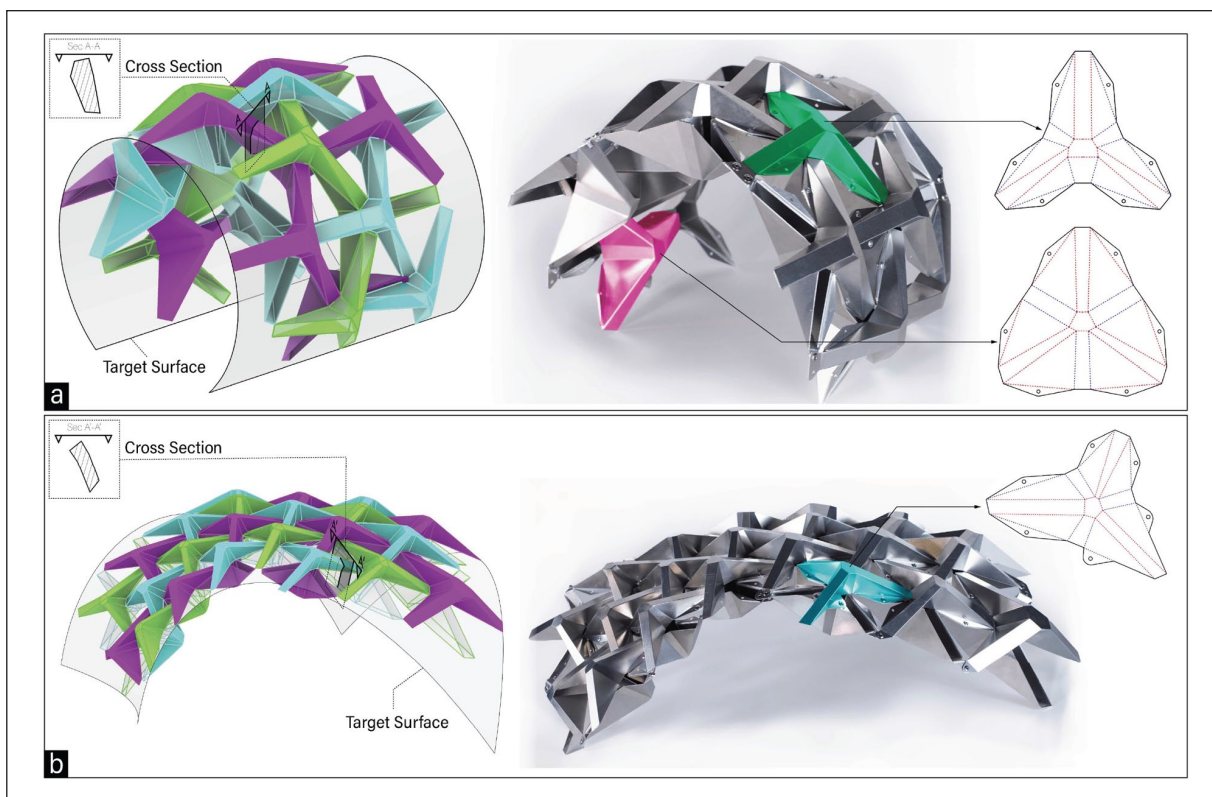


Figure 9. Results of the described method. RGB color is the calculated CAD model, gray is the manufactured prototype with some cells highlighted to see the folding pattern for the resulting unit cells. Top left shows cross sections: (a) single curvature target surface and (b) doubly curved target surface.

sets. Aligning one axis of each molecule with the cylinder's ruling direction yields a single molecule type for the upper array and one for the lower array, simplifying fabrication. The cylindrical geometry also provides greater control over molecular shape, allowing variation in the sizes of the top and bottom hexagonal faces while maintaining minimal developability error (Figure 9).

The second study model is based on a surface with freely varying curvature and concavity (Figure 9(b)). In this case, each molecule is unique, requiring careful labeling and tracking during fabrication. For both this and the previous example, the molecular scale was defined empirically in relation to the surface, and optimization parameters were adjusted iteratively to achieve an aesthetically satisfactory solution while minimizing error. As future work, a more automated pipeline could be developed, for instance by grouping molecules according to the local curvature and concavity of the surface regions they occupy.

Results

Figure 9 presents two medium-scale prototypes (0.3 m to 0.6 m): one representing a single-curvature surface, the other a doubly curved, pavilion-like structure (Figure 2). Our solver computes the top and bottom layers

independently, allowing for separate parameter control on each side of the target geometry. The highlighted cross-section illustrates how the system forms irregular hexagonal shapes that, while non-uniform, maintain structural continuity.

The prototypes were fabricated from 1.2 mm thick Aluminum 5052 sheets. The manufacturing process involved CNC milling, manual folding, and assembly using blind rivets (Figure 10). Cell outlines and crease patterns were cut with a Zünd G3 Digital Cutter L-2500 equipped with an RLM module. Folding lines were engraved to a depth of 650 microns using a 2 mm single-flute flat end mill, followed by a 100 micron finishing pass with a V-cut end mill. Final assembly was performed manually, with 1/8-inch blind rivets used for fastening.

Conclusions and future work

This work presents a method for generating reticular-like structures through folding and discrete assembly. We introduce a topological strategy for assembling Ron Resch-inspired triangular cells and describe the mechanisms that provide stiffness compared with other configurations. We develop an analytical model for the flat state of the system, which feeds numerical optimization for form-finding.

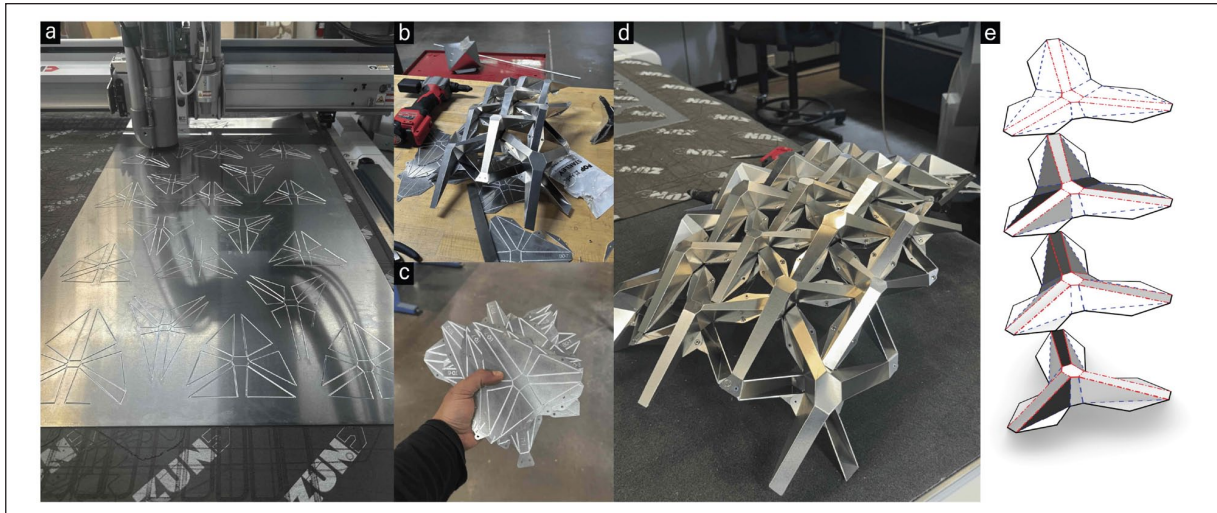


Figure 10. (a) Aluminum stock with engraved folding maps of unit cells, (b) assembly progress of the doubly curved model, (c) a handful of unit cells, (d) doubly curved model manufacturing process finished, and (e) molecule folding.

Finally, we propose manufacturability features and validate the approach by constructing two physical models.

We are aware that the work presented here, while a meaningful first step, cannot cover all types of target surfaces. The hexagonal tiling used in this work requires uniform valence-3 vertex connectivity, which cannot be preserved on arbitrary doubly-curved surfaces. Classical discretization methods accommodate Gaussian curvature by introducing pentagons at positive singularities, but pentagonal cells are incompatible with our assembly strategy. Ongoing work explores an alternative approach: discretizations that admit both valence-3 and valence-4 vertices, producing lattices of hexagons and squares rather than hexagons and pentagons. Quadrilateral cells remain compatible with our method, which would extend it to closed shells and higher-curvature geometries.

The current method is also rigid with respect to the internal topology of the tripod: crease patterns within each cell are fixed prior to optimization. In parallel, we are developing a custom solver that relaxes this constraint, allowing the internal topology of the cell to readapt during form-finding to satisfy foldability under new constraints and weights. The solver will study what configurations a cell naturally tends toward at a given curvature, with the goal of packaging this into a user-friendly web interface for designing these structures without requiring specialized software.

We are also pursuing large-scale prototypes with bolted connections and complex curvature, and investigating how geometric parameters influence structural performance, particularly cross-sectional control and bending stiffness.

We believe that this method offers a viable approach for producing affordable, transportable, and easily assembled freeform structural grids across scales. We showed, for the

regular flat configuration, that this novel modular system improves structural performance, with continuity of the second moment of inertia across the assembly acting alongside moment continuity and triaxial load redistribution at the valence-3 vertices. We discussed that its particular volume-continuous woven network morphology can provide opportunities for addressing additional functions, like insulation, heat exchange, routing of electrical systems, natural ventilation, and lighting.

This method can offer a valuable contribution to the development of structural modular folded shells at various scales not only for architectural applications but also for broader engineering and robotic use cases.

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